Multiple Coulomb-scattering in multiwire proportional chambers

Dennis Edmund Wisinski
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MULTIPLE COULOMB-SCATTERING
IN MULTIWIRE PROPORTIONAL CHAMBERS

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Presented in partial fulfillment of the requirements for the degree of

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Multiple Coulomb-Scattering in Multiwire Proportional Chambers
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The problem of multiple scattering of 100MeV pions in the wires of a multiwire proportional chamber is examined. A method of calculating the multiple scattering, using theoretical considerations, is presented. It is hypothesized that the scattering of the fraction of the incident beam that actually strikes the wires may be closely approximated by scattering the same fraction of the pions in a foil of equivalent thickness, $t_{equiv}$. This equivalent thickness is taken to be the average thickness that is encountered by a pion as it goes through a wire.

The hypothesis is tested by means of two Monte Carlo FORTRAN programs. One program simulates multiple scattering in a wire, the other simulates multiple scattering in a foil of thickness $t_{equiv}$. The salient characteristics of the programs are described and program listings are given.

Results of the Monte Carlo simulation show good agreement between the wire and the equivalent-thickness foil for large-angle single scattering, and fair agreement in the multiple and plural scattering regions. The difference between the wire multiple-scattering distribution and that of the foil is that the Gaussian portion of the wire distribution is more sharply peaked than that of the foil.

The equivalent-thickness foil is judged to be a good approximation for the wires in a multiwire proportional chamber for analysis of those experiments where scattering angles of less than a few milliradians are not observed.
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1. INTRODUCTION

Any charged particle traveling through matter undergoes many successive scatterings which change its direction and energy. The predominant type of scattering is that due to the Coulomb force. The passage of charged particles through matter has been of considerable importance in physics ever since the experiments of Rutherford, in which the nucleus of the atom was discovered in alpha-particle scattering experiments. Thomson had postulated a model of the atom in which electrons were located within a continuous distribution of positive charge. A charged particle passing through Thomson's atom would be deflected by only a small amount. It would be impossible for large-angle scattering to occur as the result of a single collision. Large-angle scattering could only be the result of many small-angle scattering events. The resulting angular distribution for multiple-scattered particles using such a model can be shown to be a Gaussian (normal) distribution. Rutherford found many more scatters of large angle than would have been expected on the basis of the Thomson model, and proposed a model of the atom which had a small positive massive nucleus surrounded by much lighter electrons. Rutherford derived a differential cross section for the elastic scattering of charged particles under the influence of a Coulomb force. This cross section was found to give an excellent description of scattering at large angles. However it was not satisfactory in calculating small-angle scattering. The Rutherford differential cross section becomes infinite at zero, and when integrated yields an infinite total cross section. And being a single-scattering formula, it does not account for multiple
scattering.

Wentzel\(^3\) first saw that multiple scattering must also be considered, and derived formulas for up to seven scattering events. Subsequently, there were many theories of varying scope and completeness proposed. A summary of the various contributions to multiple-scattering theory is given by Scott\(^4\). The multiple-scattering theories of Molière\(^5\), and Nigam, Sundaresan, and Wu\(^6\) (NSW) are generally considered to give a complete description of multiple scattering. Many further corrections and modifications have been made such as the inelastic scattering of atomic electrons (Fano\(^7\)), center-of-mass correction (Hungerford et al.\(^8\)), polarization effects (Dalitz\(^9\)), etc., but these are minor modifications. Molière's theory differs from that of Nigam, Sundaresan and Wu primarily in the treatment of the exact form of the single-scattering law. The results predicted by the theories agree closely with each other and with experimental data for all elements with atomic charge \(Z \geq 5\), even for projectile energies available in modern accelerators\(^10\).
2. THESIS PROBLEM

Multiple Scattering in Particle Detectors

In the design, execution and evaluation of any experiment involving the scattering of charged particles, the effects of multiple scattering must be taken into account. This is true of not only the target material itself, but also of intermediate particle detectors, i.e., detectors through which the particles must pass before arriving at the final detector. For the simple case where the target or detector can be treated as a uniform scattering medium of thickness \( t \), whose extent in the plane perpendicular to the beam direction is much larger than the beam diameter, the multiple scattering can be readily calculated from the Molière theory analytically, or can be simulated by a computer program employing multiple scattering distributions.

For certain types of detectors, however, this is not the case. An example is the multiwire proportional chamber (MWPC), the details of which can be found in Appendix A. The wires in this type of detector are small compared to the beam width. Most particles (about 99%) do not encounter a wire while traversing the detector. The 1% that do strike one of the wires in the wire plane undergo multiple scattering in the wire. And about 1% of these are again scattered in one of the wires in the second wire plane. The multiple scattering which occurs in these wires (which are long thin cylinders) has not been calculated analytically because of the difficulty in solving the problem for any but the simplest of geometries, namely, a layer of thickness \( t \) of homogeneous matter.

Although numerical evaluation by computer is possible by taking
the cylinder to be made of a series of layers, this is not usually
done in Monte Carlo programs that are used to simulate experiments,
because the complexity and running time of the program would be
greatly increased. What is usually done is to consider the mass of
the wires to be uniformly distributed in the plane formed by the
wires. The standard multiple scattering formulas for a layer are then
used. While this may be a sufficient approximation for experiments
where only large-angle scattering is of interest (since large-angle
scattering is single scattering which depends only on the total amount
of material present) it may not be sufficient for experiments when
data in the small-angle region are of importance.

The Proposed Solution

The ideal solution to the problem of finding an expression for
multiple scattering in wires would be an exact analytic expression,
either in closed-form or as a rapidly converging series. Unfortu-
nately this solution has not been made. The next most desirable situ-
atation would be to find an analytic formula which would approximate
the wire as an equivalent layer of some thickness $t_{\text{equiv}}$, in such a
way as to give a reasonably good approximation to the wire not only
for single (large-angle) scattering but also for multiple and plural
scattering. (The plural scattering region is that in which the parti-
cle has scattered several times at moderate angles.)

It is the contention of this thesis that the best way to approx-
nimate the wires in a MWPC for the purpose of calculating multiple
scattering, is to take the thickness of the equivalent layer to be
\[ t_{\text{equiv.}} = \frac{r}{2} \]  

where \( r \) is the radius of the wire.

**Method of Testing the Proposed Solution**

In order to check the accuracy of the proposed solution, a FORTRAN Monte Carlo program was written which simulates the passage of charged particles through a foil of thickness \( t \) and through a wire of radius \( r \). By comparing the distribution of the multiple-scattering angles in the wire of radius \( r \) to that of a foil of thickness \( t_{\text{equiv.}} \), it can be determined if this thickness gives a result close to that of the wire.
3. MOLIÈRE MULTIPLE-SCATTERING THEORY

Only the results of Molière multiple-scattering theory will be presented. The derivations are too long to be included here but can be found in detail in the Scott review article. The assumptions which are used are the standard ones:

(a) small angle approximation ($\sin \theta = \theta$, $\cos \theta = 1$, integrals from 0 to $\pi$ are replaced with ones from 0 to $\infty$);

(b) screening of nucleus by atomic electrons;

(c) no scattering of incident particles by the atomic electrons;

(d) no center-of-mass correction for heavy nuclei.

Since the incident particles will be pions (which have zero spin), no spin effects need to be considered.

The Molière multiple-scattering distributions are $F(\theta,t)$, the distribution for particles scattered into the $\theta$-direction after a thickness $t$ of material, and $f(\varphi,t)$, the projected-angle distribution, which is the projection of $F(\theta,t)$ on the $x$-$z$ plane or the $y$-$z$ plane.

Figure 1 shows the relation between these angles. These distribution functions are given by

$$F(\theta,t) = \frac{1}{2\pi} \left[ 2e^{-s^2} + \frac{1}{B} F_1(s) + \frac{1}{B^2} F_2(s) + \ldots \right], \quad (2)$$

where

$$s = \frac{\theta}{\chi eB^{\frac{1}{2}}}, \quad (3)$$

$$F_n(s) = \frac{1}{n!} \int_0^\infty \eta \, d\eta J_0(s\eta) \exp(-\frac{1}{4} \eta^4) \left( \frac{1}{4} \eta^4 \ln^2 \eta^4 \right)^n, \quad (4)$$

and

$$f(\varphi,t) = \frac{2}{\pi t} e^{-q^2} + \frac{1}{B} f_1(q) + \frac{1}{B^2} f_2(q) + \ldots, \quad (5)$$

where

$$q = \frac{\varphi}{\chi eB^{\frac{1}{2}}}, \quad (6)$$

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The quantity $B$ is defined by the transcendental equation

$$\frac{e^B}{B} = \frac{\Omega_o}{1.167},$$

where $\Omega_o = \frac{\chi_c^2}{\chi_*^2}$

The $1/e$ multiple-scattering angle is given by $\chi_c B^\frac{1}{2}$ where

$$\chi_c^2 = \left(0.396zZ\right) \frac{p}{A} \frac{T}{p^2c^2/\beta^2} \text{ (rad}^2\text{)}.$$  

And $\chi_*$ is given by

$$\chi_*^2 = \chi_o(1.13 + 3.7\alpha^2),$$

$$\chi_o = \frac{\bar{\mu}}{p\rho_o},$$

$$r_o = 0.468 \times 10^{-8} Z^{-\frac{1}{3}} \text{ (cm)},$$

$$\alpha = zZ/\sqrt{37\beta}.$$  

In the above equations $t$ is the target thickness in cm, $A$ is the atomic weight, $\rho$ is the target density in g/cm$^3$, $pc$ is the particle's momentum in MeV, $z$ is the charge of the incident particle, $Z$ is the charge of the target nucleus, and $\beta$ is the relativistic parameter $v/c$, where $v$ is the projectile's velocity.
Figure 1. Illustrating the spatial angle $\theta$ and the projected angles $\phi_x$ and $\phi_y$. 
4. CALCULATION OF THE EQUIVALENT THICKNESS

The actual situation to be investigated is the multiple scattering of π⁺ particles in a wire of radius \( r \). The energy of the incident pion beam is 100 MeV. The wire is made of tungsten and is plated with a very thin layer of gold. Since \( Z, A, \) and \( \rho \) are all about the same values for tungsten and gold, the wire will be treated as being made of tungsten only. The density for tungsten is 19.3 g/cm\(^3\). Its atomic number is \( Z = 74 \), and its atomic weight is \( A = 183.85 \) g/mole. \( z \) for the pion is 1. \( p \) and \( v \) are calculated from the energy and mass of the pion \( (m = 139.6 \) MeV/c\(^2\)). By substituting these values into equations (2) through (14) it can be seen that only an appropriate value of \( t \) is needed in equation (10) to find the multiple scattering distributions \( F(\theta,t) \) and \( f(\phi,t) \).

To determine \( t_{\text{equiv.}} \) for a wire the following argument will be used. To find the average thickness that a particle travels through the wire, replace the wire by a series of slabs that approximate the shape of the wire. (See Figure 2.) The width of the slab at height \( x_i \) is \( t_i = 2z_i \). From the relation

\[
r^2 = z^2 + x^2
gives
\[
t_i = 2\sqrt{r^2 - x_i^2}.
\]

The average value of \( t_i \) is

\[
t_{\text{avg.}} = \frac{1}{N} \sum_{i=1}^{N} t_i = \frac{1}{N} \sum_{i=1}^{N} 2\sqrt{r^2 - x_i^2}.
\]

where \( N \) is the number of slabs. From Figure 2 we see that

\[
N = 2r/\Delta x.
\]
Substitution of (18) into (17) yields

\[ t_{\text{avg.}} = \frac{1}{r} \sum_{i} \sqrt{r^2 - x_i^2} \Delta x \]  \quad (19)

Taking the limiting case for \( \Delta x \to 0 \), we write (19) as

\[ t_{\text{avg.}} = \frac{1}{r} \int_{-r}^{r} \sqrt{r^2 - x^2} \, dx \]  \quad (20)

which upon evaluation gives

\[ t_{\text{avg.}} = \frac{1}{r} \left[ \frac{1}{2} \left\{ x\sqrt{r^2 - x^2} + r^2 \sin^{-1} \left( \frac{x}{r} \right) \right\} \right]_{-r}^{r} \]  \quad (21)

or finally

\[ t_{\text{avg.}} = \frac{\pi r}{2} \]  \quad (22)

The value \( t_{\text{avg.}} \) of equation (22) is what we will take as being the equivalent thickness \( t_{\text{equiv.}} \). This value should provide the correct large-angle scattering since the total number of scatters for a large number of pions will be the same as for the wire. The small-angle multiple scattering should be improved compared to the approximation that distributes the mass of the wire uniformly over the entire wire plane, because it more closely approximates the large number of successive scatters that actually occur in the wire.

This result is expected to hold irrespective of the size of the wire since the above relationship was derived solely from geometric considerations, no wire size having been assumed. The multiple-scattering distribution of the wire should change only as the Molière distribution changes as a function of \( t_{\text{equiv.}} \).
Figure 2. Wire approximated by $N$ slabs of $t$ by $\Delta x$. 
5. MULTIPLE-SCATTERING MONTE CARLO PROGRAM

The Monte Carlo Method

The Monte Carlo method is a way of solving a problem by use of random sampling procedures, when the problem under consideration is of a statistical nature. The multiple scattering of particles in matter is just this kind of statistical phenomenon that can be accurately described by Monte Carlo simulation. The FORTRAN program which was written to test the prediction of Chapter 2, follows a particle as it progresses through the foil or wire. When the particle has emerged from the foil or wire its angle with respect to the initial direction is recorded. The process is repeated for thousands of particles and the cumulative results are printed in the form of tables and graphs. The statistical sampling is accomplished by selecting random values from the distribution functions that describe the physical quantities associated with the collisions. The method by which this is done is well known. The procedure for sampling from a normalized distribution $f(y)$ is to select a random number $r$ which is uniformly distributed between 0 and 1, and then solve the equation

$$\int_{0}^{s} f(y)dy = r \quad (23)$$

for the sample value $s$. This gives a result of the form

$$s = g(r) \quad . \quad \quad (24)$$

Design of the Program

Two programs were actually used in the work on this problem, although the differences between them were minor. Only the initial
position of the particle and the check to see if the particle is still inside the medium are different. The two programs are FOIL20 and WIRE21. They simulate multiple scattering in foils and wires respectively. The programs were written in FORTRAN and were run on the University of Montana's DECSYSTEM 20. The basic parts of the program are:

(a) preliminary calculations;
(b) Monte Carlo loop;
(c) final calculations and output;
(d) plotting subroutines.

Preliminary calculations involve the evaluation of various physical parameters such as the Thomas-Fermi screening radius, the total Coulomb collision cross section, the mean free-path, and some of the quantities from Molière's theory.

The heart of the program is the Monte Carlo loop. It begins with a pion traveling in the +Z direction (see Figure 1) at the point where it first enters the foil or the wire. To find the distance it travels before its first collision a sample value is drawn from the distribution for the free path:

\[ f(d) = \exp(-d/\bar{f}) \]  \hspace{1cm} (25)

where \( \bar{f} \) is the mean free-path, given by

\[ \bar{f} = \frac{1}{\sigma N} \]  \hspace{1cm} (26)

with \( \sigma \) given by
\[ \sigma = \pi \left( \frac{r_\alpha}{\mu} \right)^2 \left( \frac{2ze^2}{\hbar \beta} \right)^2 \]  

(27)

\( \sigma \) is the total Coulomb cross section and \( N \) is the number of atoms per unit volume of the foil or wire. \( r_\alpha \) is the Thomas-Fermi radius, which gives a measure of the screening of the nuclear Coulomb field by the atomic electrons, and \( \mu \) is a factor of order unity introduced by Nigam, Sundaresan and Wu\(^\text{13} \). The other symbols are the same as those in Chapter 3.

Three assumptions have been made in using the above distribution for the free path:

(a) the pion interacts with only one scattering center at a time;
(b) the scattering centers are randomly distributed throughout the medium and correlations among positions of different atoms are not taken into account;
(c) the pion suffers no appreciable energy loss — while traversing the medium — which would change its velocity and therefore the total cross section and mean free path.

The first two assumptions are usual in Monte Carlo calculations involving the diffusion of fast charged particles in matter\(^\text{14} \). The third can be justified for this problem by finding the energy lost by a 100MeV pion that traverses a tungsten foil of thickness 0.030316g/cm\(^2 \). Consulting a table of pion stopping power\(^\text{15} \), we find that the stopping power is about 1.3MeV-cm\(^2 \)/g. So

\[ \text{energy loss} = (1.3\text{MeV-cm}^2/\text{g})(0.030316\text{g/cm}^2) = 0.0394\text{ MeV} \quad , \]

(28)
which is negligible for 100MeV incident energy. The calculation of equation (24) for the free-path distribution is given in Appendix C.

Once the position of the next collision center has been calculated from the free path and the particle's direction, the scattering angles $\chi$ and $\Psi$ must be chosen. $\chi$ is the polar scattering-angle which is measured with respect to the direction that the pion is traveling before the collision. It is chosen from the distribution

$$f(\chi) = \frac{4 \alpha^2}{k^2 (\chi^2 + \chi_\mu^2)^2},$$

where the small-angle approximation has been used. This equation is arrived at by employing the standard first Born approximation method for the Yukawa potential\textsuperscript{16} given by

$$V(r) = \pm \left[ \frac{2Ze^2}{r} \right] \exp(-\mu r/r_\circ).$$

$\chi_\mu$ is the Born Screening angle, which is defined as

$$\chi_\mu = \mu \chi_0 = \frac{\mu}{k r_\circ} = \frac{\mu \hbar}{p r_\circ},$$

where $k$ is the reduced wavenumber of the pion:

$$k = p/\hbar.$$

Equation (29) is essentially the Rutherford single-scattering law, modified to account for the screening of the nucleus by the electrons. Calculation of equation (24) for this distribution is done in Appendix C.

$\Psi$ is the azimuthal scattering angle, and is taken - assuming the medium is isotropic and polarization is disregarded - as being distributed uniformly between 0 and $2\pi$. It is chosen by multiplying a random
number selected from a uniform distribution between 0 and 1, by 2.

Since the pion will not in general have its velocity vector oriented along one of the laboratory axes, it is necessary to change coordinate systems for each collision. The scattering must be calculated in the coordinate system defined by the pion's velocity vector, and the new position of the pion must be related to the laboratory frame. The details of this calculation are shown in Appendix B.

Once the laboratory coordinates of the pion are found, a check is made to see if the coordinates fall within the medium or outside it. If the pion is still within the medium, the process is repeated, finding a new free path and new scattering angles. When the pion emerges from the medium its multiple-scattering angle is determined by comparing its velocity vector to the lab coordinate system.

After the Monte Carlo loop has been left, the final calculations are made. The graphs are semilogarithmically scaled, and the final results are printed in tables and graphs. Various quantities used in the program are also listed.

The subroutines that are employed are physics-library subroutines that control the plotter which produces the graphs.

A complete listing of the program WIRE21 is given in Appendix D, and Appendix E shows FOIL20, a version of the program which has been modified to show the differential cross section versus the unprojected scattering-angle.
6. RESULTS AND CONCLUSIONS

Each of the Monte Carlo programs - one for the wire and one for the foil of equivalent thickness - accumulated 110 000 events. The radius of the wire used was 0.001cm, and that of the equivalent thickness of foil was 0.00157cm. The results are shown in Figures 3 and 4, with the solid curve showing the Gaussian part of each distribution. Looking at the central portions of the distributions, it is apparent that the two distributions are very close, but the one for the wire is more sharply peaked. The widths of the distributions are almost the same for scattering angles greater than 5mrad. At angles greater than 25mrad it is difficult to make a direct comparison. This is primarily due to statistical fluctuation because of the relative infrequency of the events in this region. If we use \( n^{\pm} \) as a measure of the expected fluctuation for \( n \) counts, it is clear that the distributions coincide to within this criterion for this region. The full-width at half-maximum is 10mrad for the wire and 11mrad for the foil. The 1/e angle of the wire is 6.2mrad (extrapolating from the graph), and that of the foil is 6.6mrad. The theoretical value of the 1/e angle from Molière theory is 6.44mrad. The difference between the theoretical value and the Monte Carlo value is 2.5%, which is close to what the statistical fluctuation on this part of the curve is (\( 1/3000^t = 0.018 \), or about 2%).

As a check on the large-angle scattering we compare the average number of scatters per particle for both cases. The average number was 245.7986 for the wire and 245.7842 for the foil. These results, which agree to within 0.006%, ensure that large-angle scattering will be the same for both the wire and the foil of equivalent thickness, since the
large-angle scattering depends only on the average number of scatters. The results for the small-angle multiple scattering appear to be within 10% at half-maximum, and within 6% at the 1/e angle. Since most experiments do not look at scattering at less than a few milliradians (this being the region where the unscattered beam also lies), the fact that the wire scattering-distribution is more sharply peaked than the foil's should not be of much concern. The agreement does seem to be good at moderate angles, and very good at large angles. So the equivalent-foil thickness would seem to be a useful approximation for those cases which are of most interest to experimenters.
APPENDIX A. MULTIWIRE PROPORTIONAL CHAMBERS

A multiwire proportional chamber (MWPC) is a detector which measures the x-y positions of charged particles which pass through it, by detection of the ionization produced by the particles' passage through the the gas which fills the chamber. The x-y positions are determined by two parallel planes of parallel wires, in which the wires of one plane are placed at $90^\circ$ with respect to the wires of the other, thereby forming a two-dimensional rectangular coordinate system. The electrical characteristics of such devices are discussed in the literature\textsuperscript{17}, and will not be treated here. Of more importance for the present problem is the physical structure of a typical MWPC. It is constructed as a series of closely spaced layers. The first of these is an entrance window of thin mylar. Next is a high-voltage plane, which is a sheet of aluminized mylar, followed by a wire plane. A wire plane consists of a large number (100 - 200) of independent gold-plated tungsten wires that have a diameter of 0.002cm and are spaced about 0.2cm apart. The next layer is again a high-voltage plane, followed by a second wire plane. The wires of this plane are perpendicular to the wires of the first. Another high-voltage plane follows, and finally the exit window is reached.

A mixture of argon, freon, isobutane, and propanol is pumped through the MWPC to amplify the signal. The arrangement of the layers is shown in Figure 5 along with typical spacings.
Figure 5. Schematic representation of a multiwire proportional chamber.
APPENDIX B. PARTICLE-TO-LAB COORDINATE TRANSFORMATION

Let the X, Y, and Z axes refer to the laboratory frame, and let the unit vector $\vec{U}_1$ be the direction vector associated with the particle's velocity (Figure 6). We may write this vector in terms of its Cartesian components in the laboratory frame:

$$\vec{U}_1 = (U_{1x}, U_{1y}, U_{1z}) .$$

Now if the particle undergoes a collision and is deflected through an angle $\theta$, we may represent the new direction by the unit vector $\vec{U}_1'$. We then construct a vector $\vec{D}$ which is perpendicular to $\vec{U}_1$ and which has the magnitude

$$|\vec{D}| = \sin \theta .$$

The relation between these vectors is depicted in Figure 7. Let $\vec{V}_2$ be some vector perpendicular to $\vec{U}_1$. Then the scalar product of $\vec{U}_1$ and $\vec{V}_2$ is

$$U_{1x}V_{2x} + U_{1y}V_{2y} + U_{1z}V_{2z} = 0 .$$

Since the vector $\vec{V}_2$ was arbitrary except for the above condition, we may choose $V_{2x} = 0$ and $V_{2y} = 1$. Then

$$V_{2z} = -\frac{U_{1x}}{U_{1y}} / U_{1z} .$$

Normalizing $\vec{V}_2$ we have

$$\frac{\vec{U}_2}{|\vec{V}_2|} = \left( 0, \frac{1}{\sqrt{1 + \left( \frac{-U_{1y}}{U_{1z}} \right)^2}}, \frac{-U_{1y}/U_{1z}}{\sqrt{1 + \left( \frac{-U_{1y}}{U_{1z}} \right)^2}} \right) .$$
Next form the unit vector \( \mathbf{u}^3 \) by taking the vector product of \( \mathbf{u}^1 \) and \( \mathbf{u}^2 \):

\[
\mathbf{u}^3 = \mathbf{u}^1 \times \mathbf{u}^2 = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\mathbf{u}^1_x & \mathbf{u}^1_y & \mathbf{u}^1_z \\
\mathbf{u}^2_x & \mathbf{u}^2_y & \mathbf{u}^2_z
\end{vmatrix}
\]

(B6)

\[
\mathbf{u}^3 = [(\mathbf{u}^1_y \mathbf{u}^2_z - \mathbf{u}^2_y \mathbf{u}^1_z), (\mathbf{u}^1_x \mathbf{u}^2_z - \mathbf{u}^2_x \mathbf{u}^1_z), (\mathbf{u}^1_x \mathbf{u}^2_y - \mathbf{u}^2_x \mathbf{u}^1_y)]
\]

(B7)

\( \mathbf{u}^1, \mathbf{u}^2, \mathbf{u}^3 \) form an orthonormal coordinate system for the particle.

Once this has been established, the azimuthal angle may be selected from a distribution. \( \phi \) is measured relative to the \( \mathbf{u}^2 \) axis in the \( \mathbf{u}^2-\mathbf{u}^3 \) plane (Figure 8). We may now write

\[
\mathbf{d} = \sin \theta [\cos \phi \mathbf{u}^2 + \sin \phi \mathbf{u}^3]
\]

(B8)

Note that the term in brackets has a magnitude of 1. Finally, \( \mathbf{u}^1' \) may be expressed as

\[
\mathbf{u}^1' = \mathbf{u}^1 \cos \theta + \mathbf{d}.
\]

(B9)

The change in the laboratory frame has been

\[
\Delta x = F \times \mathbf{u}^1_x',
\]

(B10)

\[
\Delta y = F \times \mathbf{u}^1_y', \quad \text{(where } F \text{ is the free path)}
\]

(B11)

\[
\Delta z = F \times \mathbf{u}^1_z'.
\]

(B12)

Knowing the new position and direction of the particle in the laboratory frame, the process may now be repeated.
Figure 6. Initial and final direction vectors.

Figure 7. Scattering angle $\theta$ and vector $\vec{D}$.

Figure 8. Azimuthal angle $\phi$. 

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APPENDIX C. INVERSION OF SAMPLE DISTRIBUTIONS

To construct formulas from which random samples may be drawn according to given distributions, we first set up equations of the form of equation (24), calculate the definite integral, normalize the resulting function, and then invert the expression to solve for the sample value $s$.

For the free path:

$$ r = C \int_{0}^{s} \exp(-x/\bar{r}) \, dx \quad . $$

Integrating the right-hand side:

$$ r = \left[ - C\bar{r} \exp(-x/\bar{r}) \right]_{0}^{s} \quad . $$

Now for $r = 1$ we have $s = \infty$, or

$$ 1 = \left[ - C\bar{r} \exp(-x/\bar{r}) \right]_{0}^{\infty} \quad . $$

So the normalization constant is $C = 1/\bar{r}$, giving

$$ r = \left[ - \exp(-x/\bar{r}) \right]_{0}^{s} \quad . $$

Evaluating (C5):

$$ r = -\exp(-s/\bar{r}) + 1 \quad . $$

Solving for $s$ yields

$$ s = -\bar{r} \ln(1-r) \quad . $$

Equation (C7) is the desired formula. Upon insertion of uniformly
distributed random numbers (between 0 and 1), it returns values which are distributed according to equation (25).

For the polar scattering-angle we write

$$ r = \int_0^s \frac{C \chi d\chi}{(\chi^2 + \chi_\mu^2)^2} $$  \hspace{1cm} (C8)

This integration is easily performed, giving

$$ r = \left[ \frac{-C}{2(\chi^2 + \chi_\mu^2)} \right]_0^s $$  \hspace{1cm} (C9)

To get the normalization constant, set $r = 1$, and $s = \infty$:

$$ 1 = \left[ \frac{-C}{2(\chi^2 + \chi_\mu^2)} \right]_0^\infty $$  \hspace{1cm} (C10)

$$ 1 = 0 + C/2 \chi_\mu^2 $$  \hspace{1cm} (C11)

The normalization constant is $C = 2 \chi_\mu^2$. Therefore

$$ r = \left[ \frac{-\chi_\mu^2}{(\chi^2 + \chi_\mu^2)} \right]_0^s $$  \hspace{1cm} (C12)

Evaluating this gives

$$ r = \frac{-\chi_\mu^2}{s^2 + \chi_\mu^2} + 1 $$  \hspace{1cm} (C13)

Solving for $s$:

$$ s = \sqrt{\frac{r}{1 - r}} \chi_\mu $$  \hspace{1cm} (C14)

Equation (C14) is the appropriate formula for sampling the polar scattering-angle distribution (equation (29)).
APPENDIX D. LISTING OF WIRE21

This is a Monte Carlo program to simulate multiple scattering in a wire of radius R.

DIMENSION MTHTXZ(0/3160), MTHTYZ(0/3160),
1GHTXZ(0/3160), GHTYZ(0/3160),
1ANGLE(0/3160), GAUSSX(3160), GAUSSY(3160)
EQUIVALENCE (MTHTXZ(3143), NHOLD), (MTHTXZ(3144), NTERM),
1(MTHTXZ(3145), NCOUNT), (MTHTXZ(3146), NSCAT),
1(MTHTXZ(3147), MHTSQ), (MTHTXZ(3148), MU),
1(MTHTXZ(3149), MHLDFP), (MTHTXZ(3150), MTBCH2),
1(MTHTXZ(3151), MFXMIN), (MTHTXZ(3152), MFXMAX),
1(MTHTXZ(3153), MA), (MTHTXZ(3154), MZ),
1(MTHTYZ(3143), NPART), (MTHTYZ(3144), KDISP),
1(MTHTYZ(3145), NCTR1), (MTHTYZ(3146), NCTR2),
1(MTHTYZ(3147), NCTR3), (MTHTYZ(3148), MCUMXZ),
1(MTHTYZ(3149), MCUMYZ), (MTHTYZ(3150), MCHICU),
1(MTHTYZ(3151), MFYMIN), (MTHTYZ(3152), MFYMAX),
1(MTHTYZ(3153), MNRO), (MTHTYZ(3154), MENERG)

DEFINE CONSTANTS AND SET INITIAL VALUES.

NCOUNT=0
NHOLD=0
HOLDFP=0.0
THTSQD=0.0
CHICUM=0.0
CUMXZ=0.0
CUMYZ=0.0
NSCAT=0
NCTR1=0

TYPE 1
1 FORMAT(' TYPE 'R' IF THIS IS A RESTART. ',$)
ACCEPT 6, RESTR
IF (RESTR .EQ. 'R') GO TO 15
GO TO 21

15 CONTINUE
OPEN (UNIT=21, FILE='FOR21.DAT', ACCESS=
1'RANDOM', MODE='BINARY', DISPOSE='SAVE', RECORD SIZE=3146)
REWIND 21
DO 2 I=0,3160
READ (21) MTHTXZ(I), MTHTYZ(I)
CONTINUE
CLOSE (UNIT=21, DISPOSE='SAVE', FILE='FOR21.DAT')
HOLDFP=FLOAT(MHLDFP)/1000000.0
T6PCH2=FLOAT(MTGH2)/1000000.0
YMU=FLOAT(MU)/1000000.0
THTSQD=FLOAT(MHTSQ)/1000000.0
CUMXZ=FLOAT(MCUMXZ)/1000000.0
CUMYZ=FLOAT(MCUMYZ)/1000000.0
CHICUM=FLOAT(MCHICU)/1000000.0
FMIN=FLOAT(MFXMIN)/1000000.0

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FXMAX=FLOAT(MFXMAX)/1000000.0
FYMIN=FLOAT(MFYMIN)/1000000.0
FYMAX=FLOAT(MFYMAX)/1000000.0
A=FLOAT(MA)/1000000.0
Z=FLOAT(MZ)/1000000.0
RHO=FLOAT(MRHO)/1000000.0
ENERGY=FLOAT(MENERGY)/1000000.0

CONTINUE
IF (NMOLD .EQ. 1) TERMIN='D'
IF (NMOLD .EQ. 0) TERMIN='T'

TYPE 5
FORMAT(' TYPE 'TEST' OR 'RUN'. ','$)
ACCEPT 7, CHECK

TYPE 3
FORMAT(' TYPE 'D' FOR DECWRITER, 'T' FOR TEKTRONIX. ','$)
ACCEPT 6, TERMIN

TYPE 17
FORMAT(' Z, A, RHO, T (IN GM/CM2), ENERGY. ','$
ACCEPT 18, Z, A, RHO, TGPCM2, ENERGY

CONTINUE

TYPE 8
FORMAT(' NTERM = ','$
ACCEPT 9, NTERM

TYPE 24
FORMAT( ' RANDOM NUMBER SEED: ' ','$
ACCEPT 9, NSEED
CALL SETRAN(NSEED)

CONTINUE

FN2T0T=DFLOAT(N2TOT)
FN3T0T=DFLOAT(N3TOT)
PI=3.1415927
K=0.001
HBARC=1.9733E-11
FINE=1.0/137.0
E=SQRT(FINE*HBARC)
BOHR=5.292E-9
ZPION=1.0
T=TGPCM2/RHO
PIMASS=139.6
ELMASS=.511004
OMEGA=ENERGY/PIMASS
BETA=SQRT(OMEGA**2+2.0*OMEGA)/(1.0+OMEGA)
GAMMA=1.0/SQRT(1.0-BETA**2)
ALPHA=(Z*ZPION*FINE)/(BETA)
AVOGAD=6.022E23
ATOMS=(RHO*AVOGAD)/A
PC=IMASS*BETA*(1.0+ENERGY/IMASS)
WAVEK=PC/HBARC
11 FORMAT(E)
TFRAD=0.885*BOHR*Z**(-1.0/3.0)
VO=1.0/(TFRAD*WAVEK)
VC=0.396*ZPI0N*Z*SQRT(RHO*Z/A)/(PC*BETA)
VA=SQRT(VO**2*(1.13+3.76*ALPHA**2))
YMU=VA/VO
VMU=VA
OMEGAO=VC**2/VA**2
VMAX=HBARC/(PC*THMRAD)
TYPE 12
12 FORMAT(' NUMBER OF PARTICLES = ','$')
ACCEPT 13, NPART
13 FORMAT(I)
TYPE 14
14 FORMAT(' DISPLAY NUMBER = ','$')
ACCEPT 13, KDISP
23 CONTINUE
IF (TERMIN .EQ. 'D ') GO TO 16
CALL INITTC120)
16 CONTINUE
C CALCULATE FP USING THE
C FIRST BORN TOTAL CROSS SECTION FOR A YUKAWA POTENTIAL, AND
C THE THOMAS-FERMI VALUE FOR THE EFFECTIVE RANGE.
SIGMA=PI*(TFRAD/YMU)**2*((2*ZPI0N*Z*E**2)/(HBARC*BETA))**2
FPM=(1.0)/(SIGMA*ATOMS)
19 CONTINUE
75 CONTINUE
TYPE 30
30 FORMAT(' JUST RESULTS?','$')
ACCEPT 31, RESANS
31 FORMAT(A3)
IF (RESANS .EQ. 'YES') GO TO 1050
C MONTE CARLO LOOP
111 DO 1001 K=1,NPART
NCOUNT=NCOUNT+1
C SPECIFY POSITION OF PION AT INITIAL COLLISION.
XCAP=(RAN(J)-0.5)*2.0*R
YCAP=0.0
ZCAP=-FPM*ALOG(1.0-RAN(J))-SQRT(R**2-XCAP**2)
PHI=0.0
THETA=0.0
U1X=0.0
U1Y=0.0
U1Z=1.0
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NSCAT = NSCAT + 1

C SELECT POLAR SCATTERING ANGLE CHI.

XRAN = RAN(J)
CHI = SQRT(XRAN / (1.0 - XRAN)) * VMU
IF (CHI, LT, VMU) GO TO 217
IF (CHI, GE, VMAX) GO TO 218
NCTR1 = NCTR1 + 1
CHICUM = CHICUM + CHI**2

CONTINUE

C FIND MAGNITUDE OF VECTOR D.
D = SIN(CHI)

C GENERATE ORTHONORMAL VECTOR BASIS U1, U2, U3.

IF (U1Z, EQ, 0.0) GO TO 220
V2Z = -U1Y / U1Z
U2X = 0.0
U2NORM = SQRT(1.0 + V2Z**2)
U2Y = 1.0 / U2NORM
U2Z = V2Z / U2NORM
GO TO 240

CONTINUE
IF (U1Y, EQ, 0.0) GO TO 230
U2X = 0.0
U2Y = 0.0
U2Z = 1.0
GO TO 240

CONTINUE
U2X = 0.0
U2Y = 1.0
U2Z = 0.0

CONTINUE
U3X = U1Y * U2Z - U2Y * U1Z
U3Y = U1X * U2Z - U2X * U1Z
U3Z = U1X * U2Y - U2X * U1Y

C SELECT AZIMUTHAL SCATTERING ANGLE PSI IN U1, U2, U3.

PSI = RAN(J) * 2.0 * PI

C FIND VECTOR D IN TERMS OF U2 AND U3.

DX = SIN(CHI) * (COS(PSI) * U2X + SIN(PSI) * U3X)
DY = SIN(CHI) * (COS(PSI) * U2Y + SIN(PSI) * U3Y)
DZ = SIN(CHI) * (COS(PSI) * U2Z + SIN(PSI) * U3Z)

C CONSTRUCT NEW U1.

U1X = U1X * COS(CHI) + DX
U1Y = U1Y * COS(CHI) + DY
U1Z = U1Z * COS(CHI) + DZ
C SELECT FREE-PATH LENGTH.

\[
FP = -\text{FPM} \cdot \text{ALOG}(1.0 - \text{RAN}(J))
\]

HOLDFP = HOLDFP + FP

C CALCULATE CHANGE IN LAB COORDINATES.

DELTAX = FP \cdot U1X
DELTAY = FP \cdot U1Y
DELTAZ = FP \cdot U1Z

C FIND NEW POSITION OF PARTICLE IN LAB FRAME.

XCAP = XCAP + DELTAX
YCAP = YCAP + DELTAY
ZCAP = ZCAP + DELTAZ

C CHECK TO SEE IF PION HAS EMERGED FROM THE FOIL.

\[
\text{SMALLR} = \text{SORT}(\text{XCAP}^2 + \text{ZCAP}^2)
\]

IF (SMALLR .GE. R) GO TO 250
IF (ABS(YCAP) .GE. 10.0) GO TO 250
GO TO 190

C NOW THAT THE PION HAS EMERGED FROM THE WIRE, DETERMINE
C THE PROJECTED ANGLES THETXZ AND THETYZ.

250 THETXZ = ATAN(DELTAX/DELTAZ)
270 IF (DELTAX) 272, 275
272 IF (DELTAZ) 277, 300
275 IF (DELTAZ) 277, 300
277 THETXZ = THETXZ + PI
280 CONTINUE

THETYZ = ATAN(DELTAY/DELTAZ)
290 IF (DELTAY) 292, 300
292 IF (DELTAZ) 297, 300
295 IF (DELTAZ) 297, 300
297 THETYZ = THETYZ + PI
300 CONTINUE

\[
\text{THTSQD} = \text{THTSQD} + \text{THETXZ}^2 + \text{THETYZ}^2
\]

CUMXZ = CUMXZ + THETXZ**2
CUMYZ = CUMYZ + THETYZ**2

C CONVERT PROJECTED ANGLES TO INDEX-INTEGERS FOR ARRAYS.

\[
\text{ITHTXZ} = ((\text{THETXZ} \times (\text{FNTOT} + 1.0)) / \text{PI} + 0.5) + \text{FNTOT} / 2
\]

\[
\text{ITHTYZ} = ((\text{THETYZ} \times (\text{FNTOT} + 1.0)) / \text{PI} + 0.5) + \text{FNTOT} / 2
\]

C RECORD THE PROJECTED ANGLES IN THEIR CORRESPONDING ARRAYS.

MTHTXZ(ITHTXZ) = MTHTXZ(ITHTXZ) + 1
MTHTYZ(ITHTYZ) = MTHTYZ(ITHTYZ) + 1

C SCALE GRAPH.

IF (FXMAX LT MTHTXZ(ITHTXZ)) FXMAX = MTHTXZ(ITHTXZ)
IF (FYMAX LT MTHTYZ(ITHTYZ)) FYMAX = MTHTYZ(ITHTYZ)
IF (MOD(KKDISP) EQ. 0) TYPE 888, K

888 FORMAT(I)

IF (TERMIN EQ. 'D') GO TO 1001
KLEAR = KLEAR + 1
IF (KLEAR EQ. 70) GO TO 890
GO TO 1001

890 CALL ERASE
KLEAR = 0

C END OF MONTE CARLO LOOP.

1001 CONTINUE

C PLOT DISTRIBUTIONS.

1050 ABEGIN = -PI*1000.0
AQUIT = PI*1000.0
AINCR = (AQUIT - ABEGIN) / FN2TOT
AN = ABEGIN
DO 1100 I = 0, N2TOT
ANGLE(I) = (AN + .02) / 2.0
AN = AN + AINCR
1100 CONTINUE

IF (TERMIN EQ. 'D') GO TO 1150
CALL ERASE

1150 GXMAX = 4.0
GYMAX = 4.0
GXMIN = AL0G10(FXMIN)
GYMIN = AL0G10(FYMIN)
ITEMPX = 0
DO 1165 I = 0, N2TOT
IF (MTHTXZ(I) GT FXMAX / EXP(1.0)) GO TO 1166
ITEMPX = I
1165 CONTINUE
1166 AN1EX = ANGLE(ITEMPX)
SDX = SQRT(AN1EX**2 / 2.0) * 2.0
ITEMPY = 0
DO 1167 I = 0, N2TOT
IF (MTHTYZ(I) GT FYMAX / EXP(1.0)) GO TO 1168
ITEMPY = I
1167 CONTINUE
1168 AN1EY = ANGLE(ITEMPY)
SDY = SQRT(AN1EY**2 / 2.0) * 2.0
AN = ABEGIN
DO 1170 I = 0, N2TOT
GAUSSX(I+1) = AL0G10(FXMAX * EXP(-AN**2 / (2.0 * SDX**2)))
GAUSSY(I+1) = AL0G10(FYMAX * EXP(-AN**2 / (2.0 * SDY**2)))
AN = AN + AINCR
1170 CONTINUE
DO 1200 I = 0, N2TOT

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GTHTXZ(I)=FLOAT(MTHTXZ(I))
GTHTYZ(I)=FLOAT(MTHTYZ(I))
IF (GTHTXZ(I) .LE. 0.0) GTHTXZ(I)=0.5
IF (GTHTYZ(I) .LE. 0.0) GTHTYZ(I)=0.5
GTHTXZ(I)=ALOG10(GTHTXZ(I))
GTHTYZ(I)=ALOG10(GTHTYZ(I))
CONTINUE
BEGINA=BEGIN/(10.0*PI)
QUITA=QUIT/(10.0*PI)
IF (TERMIN .EQ. 'D ') GO TO 1203
IF (MOIKNCOUNT .EQ. 0) GO TO 1201
GO TO 1230
1201 TYPE 1180
1180 FORMAT(' DO YOU WANT GRAPHS? ',$)
ACCEPT 1181, PLOT1
1181 FORMAT(A3)
IF (PLOT1 .EQ. 'NO') GO TO 1230
1183 CALL SETUP(1,1,4,4,BEGINA,QUITA,GXMIN,GXMAX,1)
CALL PLSPLT(ANGLE,GTHTXZ,0,N2TOT,1)
CALL SETUP(1,1,4,4,BEGINA,QUITA,GXMIN,GXMAX,1)
CALL LINPLT(ANGLE,GAUSSX,0,N2TOT,1)
CALL ERASE
CALL SETUP(1,1,4,4,BEGINA,QUITA,GYMIN,GYMAX,1)
CALL PLSPLT(ANGLE,GTHTYZ,0,N2TOT,1)
CALL SETUP(1,1,4,4,BEGINA,QUITA,GYMIN,GYMAX,1)
CALL LINPLT(ANGLE,GAUSSY,0,N2TOT,1)
CALL ERASE
TYPE 980
980 FORMAT(' DO YOU WANT GRAPHS AGAIN? ')
ACCEPT 981, GRAF
981 FORMAT(A3)
IF (GRAF .EQ. 'NO') GO TO 1230
1203 IF (CHECK .EQ. 'RUN') GO TO 1210
TYPE 1204
1204 FORMAT(' PRINT TABLE? ',$)
ACCEPT 1206, TABLE
1206 FORMAT(A3)
IF (TABLE .EQ. 'NO') GO TO 1230
1210 TYPE 1212
1212 FORMAT(/,/' ANGLE (MRAD) NO. OF EVENTS (XZ)'
DO 1230 I=1546,1596
TYPE 1220, ANGLE(I), MTHTXZ(I), MTHTYZ(I)
1220 FORMAT(7X,F7.1,13X,18X,16X,18X)
1230 CONTINUE

MHLDFP=H0LDFP*1000000
MTGCM2=TGCM2*1000000
MU=YMU*1000000
MHTSQ=HTSQ*1000000
MCUMXZ=CUMXZ*1000000
MCUMYZ=CUMYZ*1000000
MCHICU=CHICUM*1000000
MFXMIN=FXMIN*1000000
MFXMAX=FXMAX*1000000
MFYMIN=FYMIN*1000000

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MFYMAX = Fymax x 1000000
MA = A x 1000000
MZ = Z x 1000000
MRHO = RH0 x 1000000
MENERG = ENERGY x 1000000
OPEN (UNIT=21, FILE='FOR21.DAT', ACCESS= 
1' RANDOM ', MODE='BINARY', DISPOSE='SAVE', RECORD SIZE=3146)
REWIND 21
DO 1222 I=0,3160
WRITE (21) MTHTXZ(I), MTHTYZ(I)
CONTINUE
IF (CHECK .EQ. 'RUN') GO TO 983
CONTINUE
CLOSE (UNIT=21, DISPOSE='SAVE', FILE='FOR21.DAT')
TYPE 1226, NTERM, TERMIN
FORMAT(' NTERM = ', 16, ' TERMIN = ', A1, '/)
CONTINUE
IF (MOD(NCOUNT, NTERM) .EQ. 0) GO TO 1240
GO TO 111
TYPE 1300
FORMAT(' CHOOSE ONE OF THE FOLLOWING:', '/', 
1' (1) CHANGE TERMINALS AND CONTINUE.', '/', 
1' (2) CHANGE TERMINALS AND PLOT RESULTS.', '/', 
1' (3) CHANGE NTERM.', '/', 
1' (4) CONTINUE.', '/', 
1' (5) STOP.')
ACCEPT 1302, NEW
FORMAT(I)
GO TO (1308, 1309, 1311, 111, 982), NEW
GO TO 111
TYPE 1306
FORMAT(' TYPE 'GO' AFTER ATTACHING. ')
ACCEPT 1307, ATT
TYPE 1305
FORMAT(' PROGRAM SUCCESSFULLY ATTACHED.')
TYPE 3
ACCEPT 6, TERMIN
IF (TERM .EQ. 'D') NHOH=1.
IF (TERM .EQ. 'T') NHOH=0
TYPE 1309
FORMAT(' RESET NTERM?'),$
ACCEPT 1310, RESET
FORMAT(A3)
IF (RESET .EQ. 'YES') GO TO 1311
GO TO (111, 1050, 111, 111, 111), NEW
GO TO 111
TYPE 1312
FORMAT(' NTERM = '),$
ACCEPT 1302, NTERM
GO TO (111, 1050, 111, 111, 111), NEW
GO TO 111
C PRINT OUT VALUES OF INTEREST.
982 IF (TERM .EQ. 'T') CALL ERASE
TYPE 984, NCOUNT

FORMAT(' TOTAL NUMBER OF PARTICLES = ',6I)

AVGSCT=FLOAT(NSCAT)/FLOAT(NCOUNT)

TYPE 999, AVGSCT

FORMAT(' AVERAGE NUMBER OF SCATTERS = ',F)

TYPE 889, NCTR1

AVGFp=HOLDFP/FLOAT(NSCAT-1)

TYPE 996, AVGFp, FPM

AVGSCT=FLOAT(NSCAT)/FLOAT(NCOUNT)

TYPE 1350, ENERGY, RHD

TYPE 1352, T, TBPCM2

TYPE 1005, BETA, ALPHA

AVGFp=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 1350, ENERGY, RHD

AVGFP=HDLDFP/FLQAT<NGCAT-1)

TYPE 996, AVGFp, FPM

AVGFP=HDLDFP/FLQAT<NGCAT-1)
IF (NEW .EQ. 5) STOP
IF (CHECK .EQ. 'RUN') GO TO 1223
STOP
END
APPENDIX E. LISTING OF FOIL20

C THIS IS A MONTE CARLO PROGRAM TO SIMULATE MULTIPLE C SCATTERING IN A FOIL OF THICKNESS T. THE TABLE SHOWS C THE NUMBER OF EVENTS FOR 1 MILLIRADIAN INCREMENTS OF C THE UNPROJECTED SCATTERING-ANGLE THETA. THE GRAPH C SHOWS THE DIFFERENTIAL CROSS SECTION (IN BARNS) VS. THETA.

DIMENSION MTHTA(0/3160), 10THTA(0/3160), 1ANGLE(0/3160), GAUSSX(3160), TAILS(3160) EQUIVALENCE (MTHTA(3143), NHOLD), (MTHTA(3144), NTERM), (MTHTA(3145), NCOUNT), (MTHTA(3146), NSCAT), (MTHTA(3147), MTHTSQ), (MTHTA(3148), MU), (MTHTA(3149), MHLDFP), (MTHTA(3150), MTGCM2), (MTHTA(3151), MFXMAX), (MTHTA(3152), MA), (MTHTA(3153), MZ), (MTHTA(3154), NPART), (MTHTA(3155), KDISP), (MTHTA(3156), NCTR1), (MTHTA(3157), MCHICU), (MTHTA(3158), MRHO), (MTHTA(3159), MENREG)

C DEFINE CONSTANTS AND SET INITIAL VALUES.

NCOUNT=0 NHOLD=0 HOLDFF=0.0 THTS0D=0.0 CHICUM=0.0 NSCAT=0 NCTR1=0 TYPE 1

1 FORMAT( ' TYPE *R' IF THIS IS A RESTART. '*#') ACCEPT 6, RESTR IF (RESTR .EQ. 'R') GO TO 15 GO TO 21

CONTINUE OPEN (UNIT=20, FILE='FOR20.DAT', ACCESS='1', GOOD='RANDOM', MODE='BINARY', DISPOSE='SAVE', RECORD SIZE=3160) REWIND 20 DO 2 I=0,3160 READ (20) MTHTA(I)

CONTINUE CLOSE (UNIT=20, DISPOSE='SAVE', FILE='FOR20.DAT') HOLDFF=FLOAT(MHLDFP)/1000000.0 TFGCM2=FLOAT(MTGCM2)/1000000.0 YMU=FLOAT(MU)/1000000.0 THTS0D=FLOAT(MTHTSQ)/1000000.0 CHICUM=FLOAT(MCHICU)/1000000.0 FFXMAX=FLOAT(MFXMAX)/1000000.0 A=FLOAT(MA)/1000000.0 Z=FLOAT(MZ)/1000000.0 RHO=FLOAT(MRHO)/1000000.0

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EN\[\text{E}N\]=\text{FLO}\[\text{T}\text{A}\text{T}(\text{MENERG})/1000000.0\]

\text{CON}T\text{INUE}
\n\text{I}F (\text{NHOLD} .\text{E}Q. 1) \text{TERMIN}='D'
\n\text{I}F (\text{NHOLD} .\text{E}Q. 0) \text{TERMIN}='T'
\n\text{TYP}E 5
\n\text{F}ORM\text{AT}(' \text{TYP}E '\text{TEST}' \or '\text{RUN}', ',\$')
\n\text{AC}CEPT 7, \text{CHECK}
\n\text{F}ORM\text{AT}(A4)
\n\text{TYP}E 3
\n\text{F}ORM\text{AT}(' \text{TYP}E 'D' \f\text{OR} DE\text{CW}R\text{ITER}, 'T' \f\text{OR} \text{TEKTRONIX}, ',\$')
\n\text{AC}CEPT 6, \text{TERMIN}
\n\text{F}ORM\text{AT}(A1)
\n\text{I}F (\text{TERMIN} .\text{E}Q. 'D') \text{NHOLD}=1
\n\text{I}F (\text{RESTR}T .\text{E}Q. 'R') \text{GO TO 22}
\n\text{TYP}E 17
\n\text{F}ORM\text{AT}(' Z, A, RHO, T (IN GM/CM2), ENERGY, ',\$')
\n\text{AC}CEPT 18, Z, A, RHO, T\text{GFCM2}, ENERGY
\n\text{F}ORM\text{AT}(5E)
\n\text{CON}T\text{INUE}
\n\text{TYP}E 8
\n\text{F}ORM\text{AT}(' NTER M = ',\$')
\n\text{AC}CEPT 9, NTERM
\n\text{F}ORM\text{AT}(I)
\n\text{TYP}E 24
\n\text{F}ORM\text{AT}(' R\text{ANDOM NUMBER SEED: ',\$')
\n\text{AC}CEPT 9, NSEED
\n\text{C}ALL \text{SE}\text{TRAN}(\text{NSEED})
\n\text{K}\text{LEAR}=0
\n\text{FXMIN}=1.0
\n\text{DIV2}=24.67964
\n\text{N2TOT}=3142
\n\text{C}ON\text{T\text{INUE}
\n\text{F}N2TOT=DF\text{LOAT}(N2TOT)
\n\text{PI}=3.1415927
\n\text{HBAR}=1,9733E-11
\n\text{FINE}=1.0/137.0
\n\text{BARNS}=1.0E424
\n\text{E}=S\text{QRT}(\text{FINE}*\text{HBARC})
\n\text{BOHR}=5.292E-9
\n\text{ZPI0N}=1.0
\n\text{T}=T\text{GFCM2}/RHO
\n\text{PIMASS}=139.6
\n\text{ELM}\text{ASS}=5.11004
\n\text{OMEGA}=\text{EN}\text{ERGY}/\text{PIMASS}
\n\text{BETA}=S\text{QRT}(\text{OMEGA}**2+2.0*\text{OMEGA})/(1.0+\text{OMEGA})
\n\text{GAMMA}=1.0/S\text{QRT}(1.0-\text{BETA}**2)
\n\text{ALPHA}=(Z*ZPI0N*F\text{INE})/(\text{BETA})
\n\text{AVOGAD}=6.022E23
\n\text{ATOMS}=(\text{RHO}*\text{AVOGAD})/A
\n\text{FC}=\text{PIMASS}\text{BETA}*(1.0+\text{ENERGY}/\text{PIMASS})
\n\text{WAVEK}=\text{FC}/\text{HBAR}
\n\text{F}ORM\text{AT}(E)
\n\text{TFRAD}=0.885*\text{BOHR}Z**(-1.0/3.0)
\n\text{VO}=1.0/(\text{TFRAD}WAVEK)
\n\text{VC}=0.396*ZPI0N*Z*\text{SQR\text{T}(RHO*T/A)/(FC}\text{BET})
\n\text{VA}=S\text{QRT(VO}**2*(1.13+3.76*\text{ALPHA}**2))
\( \text{YMU} = \frac{V_A}{V_0} \)
\( \text{VMU} = V_A \)
\( \text{OMEGA} = \frac{V_C**2}{V_A**2} \)
\( \text{ATMRAD} = (1.40E-13) A**(1.0/3.0) \)
\( \text{VMAX} = HBARC/(PC*ATMRAD) \)

12 FORMAT( ' NUMBER OF PARTICLES = ', $ )
13 ACCEPT 13, NPART
14 FORMAT( ' DISPLAY NUMBER = ', $ )
15 CONTINUE
16 CALL INIT(120)

C CALCULATE \( F_P \) USING THE FIRST BORN
C TOTAL CROSS SECTION FOR A YUKAWA POTENTIAL, AND
C THE THOMAS-FERMI VALUE FOR THE EFFECTIVE RANGE.

\[ \text{SIGMA} = \pi \left( \frac{\text{TFRAD}}{\text{YMU}} \right)^2 \left( \frac{2 \times Z \times \text{PI} \times \text{N} \times \text{E}^2}{1 \times \text{HBARC} \times \text{BETA}} \right)^2 \]

\[ \text{FPM} = \frac{1.0}{\text{SIGMA} \times \text{ATOMS}} \]

19 CONTINUE

75 CONTINUE

30 FORMAT( ' JUST RESULTS?', $ )
31 ACCEPT 31, RESANS
32 IF (RESANS .EQ. 'YES') GO TO 1050

C MONTE CARLO LOOP

111 DO 1001 K=1,NPART
112 NCOUNT=NCOUNT+1

C SPECIFY POSITION OF PION AT INITIAL COLLISION.

\[ \text{XCAP} = 0.0 \]
\[ \text{YCAP} = 0.0 \]
\[ \text{ZCAP} = -\text{FPM} \times \text{ALOG}(1.0-\text{RAN(J)}) \]
\[ \text{U1X} = 0.0 \]
\[ \text{U1Y} = 0.0 \]
\[ \text{U1Z} = 1.0 \]

190 NSCAT=NSCAT+1

C SELECT POLAR SCATTERING ANGLE CHI.

217 XRN=\text{RAN(J)}
218 CHI=\text{SQRT}((\text{XRN}/(1.0-\text{XRN}))) \times \text{VMU}
219 IF (CHI .LT. \text{VMU}) GO TO 217
220 IF (CHI .GE. \text{VMAX}) GO TO 218
221 NCTR1=NCTR1+1
222 CHICUM=CHICUM+CHI**2
223 CONTINUE

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C FIND MAGNITUDE OF VECTOR D.

\[ D = \sin(\chi) \]

C GENERATE ORTHONORMAL VECTOR BASIS U1, U2, U3.

IF (U1Z .EQ. 0.0) GO TO 220

\[ V2Z = - U1Y / U1Z \]
\[ U2X = 0.0 \]
\[ V2NORM = \sqrt{1.0 + V2Z^2} \]
\[ U2Y = 1.0 / V2NORM \]
\[ U2Z = V2Z / V2NORM \]
GO TO 240

220 CONTINUE
IF (U1Y .EQ. 0.0) GO TO 230

\[ U2X = 0.0 \]
\[ U2Y = 0.0 \]
\[ U2Z = 1.0 \]
GO TO 240

230 CONTINUE

\[ U2X = 0.0 \]
\[ U2Y = 1.0 \]
\[ U2Z = 0.0 \]

240 CONTINUE

\[ U3X = U1Y \times U2Z - U2Y \times U1Z \]
\[ U3Y = U1X \times U2Z - U2X \times U1Z \]
\[ U3Z = U1X \times U2Y - U2X \times U1Y \]

C SELECT AZIMUTHAL SCATTERING ANGLE PSI IN U1, U2, U3.

\[ \psi = \text{RAN}(J) \times 2.0 \times \pi \]

C FIND VECTOR D IN TERMS OF U2 AND U3.

\[ \begin{align*}
D_X &= \sin(\chi) \times (\cos(\psi) \times U2X + \sin(\psi) \times U3X) \\
D_Y &= \sin(\chi) \times (\cos(\psi) \times U2Y + \sin(\psi) \times U3Y) \\
D_Z &= \sin(\chi) \times (\cos(\psi) \times U2Z + \sin(\psi) \times U3Z) 
\end{align*} \]

C CONSTRUCT NEW U1.

\[ \begin{align*}
U1X &= U1X \times \cos(\chi) + D_X \\
U1Y &= U1Y \times \cos(\chi) + D_Y \\
U1Z &= U1Z \times \cos(\chi) + D_Z 
\end{align*} \]

C SELECT FREE-PATH LENGTH.

\[ \begin{align*}
FP &= \text{FFM} \times \text{ALOG}(1.0 - \text{RAN}(J)) \\
\text{HOLDFP} &= \text{HOLDFP} + FP 
\end{align*} \]

C CALCULATE CHANGE IN LAB COORDINATES.

\[ \begin{align*}
\Delta X &= FP \times U1X \\
\Delta Y &= FP \times U1Y \\
\Delta Z &= FP \times U1Z 
\end{align*} \]
C FIND NEW POSITION OF PARTICLE IN LAB FRAME.
    XCAP=XCAP+DELTAX
    YCAP=YCAP+DELTAY
    ZCAP=ZCAP+DELTAZ

C CHECK TO SEE IF PION HAS EMERGED FROM THE FOIL.
    IF (ZCAP.GE.T) GO TO 250
    IF (ZCAP.LE.0) GO TO 250
    GO TO 190

C NOW THAT THE PION HAS EMERGED FROM THE FOIL, DETERMINE C THE ANGLE THETA.

250    DELTAR=SQRT(DELTAX**2+DELTAY**2+DELTAZ**2)
    THETA=ACOS(DELTAZ/DELTAR)
    IF (THETA .GT. VMAX) GO TO 300
    THTSQD=THTSQD+THETA**2

C CONVERT THE ANGLE THETA INTO AN ARRAY INDEX.

300    ITHTA=(THETA*FN2TOT)/PI+0.5

C TALLY THE ANGLE.
    MTHTA(ITHTA)=MTHTA(ITHTA)+1

C SCALE GRAPH.
    IF (FXMAX.LT.MTHTA(ITHTA)) FXMAX=MTHTA(ITHTA)
    IF (MOD(K,KDISP) .EQ. 0) TYPE 888, K

888    FORMAT(I)
    IF (TERMIN .EQ. 'D') GO TO 1001
    KLEAR=KLEAR+1
    IF (KCLEAR .EQ. 70) GO TO 890
    GO TO 1001

890    CALL ERASE
    KCLEAR=0

C END OF MONTE CARLO LOOP.

1001   CONTINUE

C PLOT DISTRIBUTIONS.

1050   ABEGIN=0.0
    AQUIT=PI*1000.0
    AINCR=(AQUIT-ABEGIN)/FN2TOT
    AN=ABEGIN
    DO 1100 I=0,N2TOT
    ANGLE(I)=AN
    AN=AN+AINCR

1100   CONTINUE
    IF (TERMIN .EQ. 'D') GO TO 1150
    CALL ERASE
GMAX=8.0
GMIN=3.0
CONST=BARNES/(ATOMS*T)
AN=AN+AINCR
ANR=AN/1000.0
DO 1200 1=1,N2TOT
   GTHTA(I)=FLOAT(MTHTA(I))
   IF (GTHTA(I) .LE. 0.0) GTHTA(I) =0.000001
   GTHTA(I)=(GTHTA(I)*CONST*1000.0)/(1(2.0*PI*FLOAT(NCOUNT)*SIN(ANR))
   GTHTA(I)=ALOG10(ABS(GTHTA(I)))
   AN=AN+AINCR
   ANR=AN/1000.0
1200 CONTINUE
ITEMPX=0
AN=ABEGIN
ANR=AN/1000.0
FXMAX=10.0**GTHTA(1)
DO 1165 I=N2TOT,1,-1
   IF (GTHTA(I) .GT. ALOG10(FXMAX/EXP(1.0))) GO TO 1166
ITEMPX=I
1165 CONTINUE
1166 AN1EX=ANGLE(ITEMPX)
   SDX=SQRT(AN1EX**2/2.0)
   AN=ABEGIN
   AN=AN+AINCR
   ANR=AN/1000.0
   DO 1170 I=1,N2TOT
       GAUSSX(I) =ALOG10(FXMAX*EXP(-AN**2/(2.0*SDX**2)))
       TAILS(I) =ALOG10(ALPHA**2*BARNES/(4.0*WAVEK*((2*SIN(ANR/2.0)**4))
       AN=AN+AINCR
   ANR=AN/1000.0
1170 CONTINUE
BEGINA=ABEGIN/(10.0*PI)
QUITA=AQUIT/(10.0*PI)
IF (TERMIN .EQ. 'D') GO TO 1203
IF (MOD(NCOUNT,NTERM) .EQ. 0) GO TO 1201
GO TO 1230
1201 TYPE 1180
1180 FORMAT( ' DO YOU WANT GRAPHS? ',$ )
   ACCEPT 1181, PLOT1
1181 FORMAT(A3)
   IF (PLOT1 .EQ. 'NO') GO TO 1230
   CALL SETUP(1,1,4,5,BEGINA,QUITA,GXMIN,GMAX,1)
   CALL PLSPLT(ANGLE,GTHTA,1,N2TOT,1)
   CALL DSHPLT(ANGLE,TAILS,0,N2TOT,1)
   CALL LINPLT(ANGLE,GAUSSX,0,N2TOT,1)
   CALL ERASE
   TYPE 980
980 FORMAT( ' DO YOU WANT GRAPH AGAIN? ')
   ACCEPT 981, GRAF
981 FORMAT(A3)
   IF (GRAF .EQ. 'YES') GO TO 1183
   IF (TERMIN .EQ. 'T') GO TO 1230
1203 IF (CHECK .EQ. 'RUN') GO TO 1210
TYPE 1204
FORMAT(‘PRINT TABLE? ‘, ‘$’)
ACCEPT 1206, TABLE
1206 FORMAT(A3)
IF (TABLE .EQ. ‘NO’) GO TO 1230
1210 TYPE 1212
1212 FORMAT(‘//,’ ANGLE (MRAD) NO. OF EVENTS (XZ)
1’’)
DO 1230 I=1,100
1220 TYPE 1220, ANGLE(I), MTHTA(I)
1230 CONTINUE
MHLDFP=HOLDFF*1000000
MTGCM2=TPC2M2*1000000
MU=YMU*1000000
MTHTSQ=THTSQ*1000000
MCHICUM=CHICU*1000000
MFXMAX=FXMAX*1000000
MA=A*1000000
MZ=Z*1000000
MRH0=RH0*1000000
MENERG=ENERGY*1000000
OPEN (UNIT=20, FILE=’FOR20.DAT’‘, ACCESS=
1’’ RANDOM’, ‘MOD=’ BINARY’, ‘DISPOSE=’ SAVE’, ‘RECORD SIZE=3160)
REWIND 20
DO 1222 I=0,3160
1222 WRITE (20) MTHTA(I)
CONTINUE
IF (CHECK .EQ. ‘RUN’) GO TO 983
1223 CONTINUE
CLOSE (UNIT=20, DISPOSE=’SAVE’, FILE=’FOR20.DAT’)
1226 TYPE 1226, NTERM, TERMIN
1235 CONTINUE
IF (MOD(NCOUNT, NTERM) .EQ. 0) GO TO 1240
1240 TYPE 1300
1300 FORMAT(‘CHOOSE ONE OF THE FOLLOWING:’,
1’‘ (1) CHANGE TERMINALS AND CONTINUE.’,
1’‘ (2) CHANGE TERMINALS AND PLOT RESULTS.’,
1’‘ (3) CHANGE NTERM.’,
1’‘ (4) CONTINUE.’,
1’‘ (5) STOP.’)
ACCEPT 1302, NEW
1302 FORMAT(I)
GO TO (1308, 1308, 1311, 111, 982), NEW
GO TO 111
1308 TYPE 1306
1306 FORMAT(‘TYPE ’’GO’’ AFTER ATTACHING.’)
ACCEPT 1307, ATT
1307 FORMAT(A2)
TYPE 1305
1305 FORMAT(‘PROGRAM SUCCESSFULLY ATTACHED.’)
1300 TYPE 3
ACCEPT 6, TERMIN
IF (TERMIN .EQ. ‘I’) NHOHD=1
IF (TERMIN .EQ. 'T') NHOLD=0

TYPE 1309

FORMAT('RESET NTERM?',$)
ACCEPT 1310, RESET

TYPE 1310 FORMAT(A3)

IF (RESET .EQ. 'YES') GO TO 1311
GO TO (111,1050,111,111,111), NEW
GO TO 111

TYPE 1311

FORMAT(' NTERM = ',$)
ACCEPT 1302, NTERM
GO TO (111,1050,111,111,111), NEW
GO TO 111

C PRINT OUT VALUES OF INTEREST.

IF (TERMIN .EQ. 'T') CALL ERASE

TYPE 984, NCOUNT

FORMAT('TOTAL NUMBER OF PARTICLES = ',AI)

AVGSCT=FLOAT(NSCAT)/FLOAT(NCOUNT)

TYPE 999, AVGSCT

FORMAT('AVERAGE NUMBER OF SCATTERS = ',F)

TYPE 889, NCTR1

AVGFP=HOLDFP/FLOAT(NSCAT-1)

TYPE 996, AVGFP, FPM

FORMAT('AVGFP = ',1PE14.7,' FPM = ',1PE14.7)

TYPE 1350, ENERGY, RHO

TYPE 1351, Z, A

FORMAT(' Z = ',F,' A = ',F)

TYPE 1352, T, TGPCM2

FORMAT(' T = ',F,' TGPCM2 = ',F)

TYPE 1005, BETA, ALPHA

FORMAT(' BETA = ',F,' ALPHA = ',1PE14.7)

TYPE 1006, PC, WAVEK

FORMAT(' PC = ',F,' WAVEK = ',1PE14.7)

TYPE 1007, TFRAD, VO

FORMAT(' TFRAD = ',1PE14.7,' VO = ',1PE14.7)

TYPE 1008, SIGMA, VA

FORMAT(' SIGMA = ',1PE14.7,' VA = ',1PE14.7)

TYPE 1400, VC, OMEGA0

FORMAT(' VC = ',1PE14.7,' OMEGA0 = ',0PF)

TYPE 1401, VMU, YMU

FORMAT(' VMU = ',1PE14.7,' YMU = ',0PF12.7)

TYPE 1403, VMAX, ATMRAD

FORMAT(' VMAX = ',1PE14.7,' ATMRAD = ',1PE14.7)

B=1.153+2.583*ALOG10(OMEGA0)

TYPE 1418, B

FORMAT(' B = ',F12.7)

THETMS=SQRT(B*VC**2)

THTRMS=SQRT(THTRSQD/FLOAT(NCOUNT))

TYPE 1419, THTRMS, THETMS

FORMAT(' THTRMS = ',1PE14.7,' THETMS = ',1PE14.7)

CHIRMS=SQRT(CHICUM/FLOAT(NCTR1))

AVSCAT=ATOMS*SIGMA*T

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THTMOL = SQRT(4.0*ALOG(183.0*Z**(-1.0/3.0))/B)*THETMS

TYPE 1427, CHIRMS, THTMOL

FORMAT( ' CHIRMS = ', 1PE14.7, ' THTMOL = ', 1PE14.7)
THTAVG = CHIRMS*SQRT(AVSCAT)

TYPE 1429, ANIEX

FORMAT( ' ANIEX = ', 1PE14.7)

TYPE 1435, THTAVG

FORMAT( ' THTAVG = ', 1PE14.7)
DIFFER = ((THTAVG-THTMOL)/THTMOL)*100.0

TYPE 1437, DIFFER

FORMAT( ' EST. DIFF. FROM MOLIERE THEORY : ', F7.2, ' %.' )

TYPE 1438, ADIFF

FORMAT( ' ACT. DIFF. FROM MOLIERE THEORY : ', F7.2, ' %.' )

IF (NEW .EQ. 5) STOP
IF (CHECK .EQ. 'RUN') GO TO 1223
STOP
END
REFERENCES

1. E. Rutherford, Phil. Mag. 21, 669 (1911).


11. Scott, op. cit.


