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Three Strikes

Tom O’Brien¹ and Marianne Smith²

Abstract
The first part of the paper describes and comments upon three aspects of the back-to-basics movement: the make-up and mindset of the National Mathematics Advisory Panel (NMP), the movement’s history in California, and recent “grassroots” activities of the movement in the state of Washington. The second part reports and comments on the principal findings of the NMP report.

Keywords: Mathematics education reform; Back to basics movement; California math reform; the National Mathematics Advisory Panel (NMP); policy

The first part of this article was written in advance of the March 2008 release of the final report of the National Math Panel. The observations provide a context for viewing the Panel’s work and the various responses to the final report.

In recent years there have been at least three concerted assaults against a problem-solving oriented approach to mathematics education, undermining the Principles and Standards of School Mathematics (PSSM) of the National Council of Teachers of Mathematics and the mathematics education programs and initiatives of the National Science Foundation.


Media coverage of the Panel has ranged from The Washington Post to The New York Times, and these news articles have been posted on Internet web sites, along with a variety of blog commentaries.

The intent of the report is to offer definitive advice leading to competence in algebra and readiness for higher levels of mathematics in terms of “conceptual knowledge and skills, learning processes, instructional practices, teachers and assessment.” The final report will contain recommendations and “take-away” messages.

From the draft version of the document available to the public in early December 2007, it

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appears that everything is wrong with current mathematics education in the US, and much
must be done because we are losing the achievement test score competition to countries in
Asia and Europe.

The findings of the Panel are likely to emphasize the importance of learning the “classic”
algorithms and procedures of arithmetic and algebra. The assumption is almost certain to be
that a back-to-basics approach will lead to improvement. Given the writings and
presentations of Panel members, there is likely to be little support for the use of technology
in the learning process, save the recommendation that software can be helpful in drill and
practice to master procedural skills. There will be a subtle disparagement of the value of
using “real world” problem contexts, as well as a skeptical view of the value of integrated
high school mathematics curricula.

In considering the mindset of many Panel members, it is worth bearing in mind that:

1. In 1991, almost a generation ago, a study by Educational Testing Service showed that 88.5
per cent of American thirteen year olds owned a calculator. [Educational Testing Service,
International Assessment of Educational Progress, Report 13, Background Questions and
Proficiency Scores for all Populations, 1992.]

2. The graphics calculator was invented more than two decades ago—1985.
[http://www.math.ohio-state.edu/~waitsb/inventor.html]

3. Present-day computers can perform about 500 trillion calculations in one second.

The panel is likely to base its findings on what they call “legitimate” research, which is
narrowly defined to include only those “scientific” studies conducted using a clinical trial
method of random assignment of treatments over experimental groups and control groups.
Many studies reviewed by the National Research Council over the past 10 years have shown
what is known and regarded as essential in mathematics teaching and learning; it is a good
bet that the Panel will largely choose to ignore or to cherry pick among these studies. [See
http://www.nap.edu/openbook.php?isbn=0309069955]

The panel (13 men, 6 women) consists largely of partisan appointees from the back-to-basics
camp, many of whom have no background in school mathematics, limited and/or no
experience in teaching children, or both. Thus it is a reasonable guess that they have little
understanding of the spirit of NCTM or NSF priorities, not to say anything of those of the
classroom teacher.

When the panel was first appointed in 2006, Education Week predicted, “The panelists’
backgrounds suggest they will favor a particular approach to teaching math—generally
speaking, one that stresses the need for drill and practice in basic computation at early grade
levels, at the expense of problem solving.” (Cavanagh, 2006). [“Some Worry About Potential
Bias on the National Math Panel,” EdWeek, May 19, 2006, By Sean Cavanagh] “Similar
charges of bias dogged the National Reading Panel, formed in 1997, which administration
officials claimed as a model for the math group,” (Ed Week, 2006). “ ("White House
Suggests Model Used in Reading To Elevate Math Skills,” Ed Week, Feb. 15, 2006.) The federally championed reading program has had a history of corruption and cronyism. [Justice Dept. Is Asked To Investigate Reading Plan, New York Times, By Diana Jean Schemo, April 21, 2007. [See http://select.nytimes.com/search/restricted/article?res=FA0C16FB3D5A0C728EDDAD0894DF404482]

Yet another question might be raised concerning the selection of panel members: it is not clear that mathematically knowledgeable people from science, industry, technology and the arts were represented. Many members of the panel are academics, the sorts who have intimidated generations of math students. Where were leaders who know about the real world, the economy, the future, applications of mathematics in science and technology? Perhaps most important, where were leaders who know about children and their ways of thinking and learning and mathematizing?

To their credit, the Department of Education has published transcripts of the meetings, including public commentary by non-Panel members. Transcripts are available on the NMP web site. [See http://www.ed.gov/about/bdscomm/list/mathpanel/meetings.html]

Perhaps most important of all, the transcripts contain Public Comment presentations by experts who were not part of the Panel. These provide interesting and revealing news, insights and suggestions can be read in full at the DoE website. The public comments—not always reflecting a back-to-basics viewpoint—often shed more light on important issues than the formal transcript of the Panel members’ deliberations.

What follows are excerpts from the public’s contributions. They often encourage the Panel to consider a mathematics curriculum that is more than a focus on “basic skills.”

Patrick Thompson, Professor of Mathematics Education, (Chicago meeting, April 20, 2007), brings challenges to the Panel involving ideas that are likely to be new to them.

[T]he Panel has the significant task of responding to a list of charges that take “skills” as the primary component of mathematics learning when the notion of skill itself is hardly well defined. Do you take “skill” to mean a child’s ability to perform reliably a procedure when told to perform that procedure? Or, do you take “skill” to mean a child’s ability to have developed sufficient knowledge and appropriate flexibility of thought to solve most problems of a particular genre of problems, even those that might have subtle and nuanced differences from any the students might have seen?

For the same reason, one cannot simply look to “research” to answer the question of what policies the nation should follow in preparing students for algebra. Which algebra? Push-button algebra or, as Kaput calls it, the algebra of progressive generalization and symbolization? The two entail different philosophic and intellectual commitments for those who embrace them, and they entail different expectations for students’ learning and teachers’ knowledge at every grade.

Mathematician Sol Garfunkel pleaded for the Panel’s report to be different from what he anticipates.
My comment to this Panel is don’t . . . write the report that we all expect to come out of this Panel, because I think it will set back mathematics education for a number of years. Don’t write a report that says there is a lot we don’t know . . . and [that] until that research is complete, we should stop innovation in curriculum development, except if we adopt something like the Singapore Program, and that we should cut off funding for that curriculum development, we should cut off funding for the National Science Foundation. I suspect that that’s what this report will eventually say and it’s a terrible mistake.

http://mathpanelwatch.blogspot.com/

Mathematics educator Susan Friel addressed the development of mathematical thinking in grades K-5. Her paper included a noteworthy and up-to-date bibliography (Chapel Hill meeting). The bibliography is welcome, especially in jurisdictions where the back-to-basics leaders have marginalized anything connected with the National Council of Teachers of Mathematics.

http://www.ed.gov/about/bdscomm/list/mathpanel/2nd-meeting/presentations/index.html

Classroom teacher Holly Concannon testified about her rescue of a mathematically damaged young girl and then went on to list noteworthy parallel school district successes in the Boston public schools. The curriculum leading to these successes was the reform project (i.e., NSF-supported) TERC’s Investigations in Number, Data and Space.

I am proud to share with you today the gains we have made in math. In 1999, our school had devastating results on the statewide tests. Fifty-four percent 54% of our students landed in the warning category. Six years later, we have just 9% in that same category. The number of students achieving advanced or proficient rose 32 percentage points. These statistics have given the Murphy [School] great reason to celebrate. However, we are not the only school worthy of the celebration. The Boston Public School District as a whole is making progress. Early this year, we made national headlines for having the greatest gains in our NAEP scores, among 11 major urban districts. [Cambridge meeting]

http://www.ed.gov/about/bdscomm/list/mathpanel/3rd-meeting/presentations/concannon.holly.pdf

From Randy Harter, Mathematics Specialist, Buncombe County Schools, Asheville, North Carolina.

In 2001, the Mathematics Learning Study Committee stated in Adding It Up that “Mathematics learning has often been more a matter of memorizing than of understanding.” My concern is that our longstanding traditions and culturally based instructional practices and the unbalanced emphasis on mathematics as procedures in most K-8 classrooms in this country have inhibited the development of reasoning and problem solving. For most students that come through this system, the result has been that mathematics is merely a set of procedures. A significant study by Jo Boaler, now at Stanford, came to a similar conclusion for students in England. She said, “Students thought that success in mathematics involved learning, rehearsing, and memorizing standard rules and procedures. … They did not regard mathematics to be a thinking
subject.” One student’s comment was typical, “In maths you have to remember, in other subjects, you can think about it.”

[Chapel Hill meeting] [See Note 1.]
http://www.ed.gov/about/bdscomm/list/mathpanel/2nd-meeting/presentations/index.html

Strike 2. The origins of the most recent swing of the pendulum toward back-to-basics in mathematics education has been documented in books and articles about the trajectory of education policy in the state of California during the 1990s.

By 1999, changes were made in the state’s mathematics framework and academic standards, in teacher professional development programs, as well as to textbook adoption guidelines. The changes were made under very controversial circumstances. [See Fraser's Panel testimony below.] The rigidity of these changes significantly affected professional development providers, who must sign a loyalty oath—an agreement to follow the back-to-basics California state standards.

The California back-to-basics movement has since metastasized through the years to Massachusetts, New York, Missouri, Washington, New Jersey and Utah.

Despite claims made by California back-to-basics leadership that their Standards are “world-class” and “rigorous,” 2007 data show that only 23% of California students are proficient in Algebra I by the end of high school and NAEP data showed 30 and 24 per cent of pupils proficient at grades 4 and 8 respectively. The California grade 4 NAEP results were higher than only 1 of 52 states and other jurisdictions and the grade 8 results were higher than only 4 of 52 jurisdictions. [http://www.cde.ca.gov/ta/tg/nr/caresults.asp]

Testimony by Sherry Fraser at the Palo Alto meeting on November 6, 2006 shed light on many unacknowledged aspects of the decade-long back-to-basics movement in California.

… It was at the end of that decade that the National Council of Teachers of Mathematics released their Curriculum and Evaluation Standards for School Mathematics (1989). Contrary to what you hear today, they were widely accepted and endorsed. This set of standards had the potential to help the American mathematics educational community begin to address the problems articulated throughout the 1980’s.

Shortly after publication, the National Science Foundation began funding the development of large scale, multi-grade instructional materials in mathematics to support the realization of the NCTM Standards in the classroom. Thirteen projects were funded. Each of the projects included updates in content and in the context in which mathematics topics are presented. Each also affected the role of the teacher. Each has been through rigorous development that included design, piloting, redesign, field-testing, redesign, and publication. This amount of careful development and evaluation is rarely seen in textbook production.

… These NSF projects were developed to address the crisis in mathematics education. They did not cause the problem; they were the solution to the problem. Their focus went
beyond memorizing basic skills to include thinking and reasoning mathematically.

… These model curriculum programs show potential for improving school mathematics education. When implemented as intended, research has shown a different picture of mathematics education to be more effective.

… A study by the American Association for the Advancement of Science (AAAS) evaluated 24 algebra textbooks for the potential to help students understand algebra and …, the NSF-funded curriculum programs rated at the top of the list. And in 2004 the National Academy of Sciences released a book, On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Programs, which looked at the evaluation studies for the thirteen NSF projects and six commercial textbooks. Based on the 147 research studies accepted it is quite clear which curriculum programs have promise to improve mathematics education in our country. They are the NSF-funded curriculum projects.

You might be asking yourself why hasn’t mathematics education improved if we have all this promising data from these promising programs? Let me use California as an example.

In 1997 California was developing a set of mathematics standards for K-12. A State Board member hijacked the process. She gave the standards, which had been developed through a public process, to a group of four mathematicians to fix. She wanted California’s standards to address just content and content that was easily measurable by multiple-choice exams. The NCTM standards, which the original California standards were based on, were banned and a new set of California standards was adopted instead. This new set punished students who were in secondary integrated programs and called for Algebra 1 for all 8th grade students, even though the rest of the world, including Singapore, teaches an integrated curriculum in 8th grade and throughout high school. The four mathematicians and a few others called California’s standards “world class”. But saying something is world class doesn’t make it so. In fact, we now have data to show these standards haven’t improved mathematics education at all. Most of California’s students have had all of their instruction based on these standards since they were adopted almost ten years ago. Yet, if you go to the California Department of Education’s website on testing and look at the 2006 data you will find that only 23% of students are proficient in Algebra I by the end of high school, a gain of 2 points over four years….

… Why, then, do you read in newspapers about how terrible the mathematics programs developed in the 1990’s are and how successful California is? It has to do with an organization called Mathematically Correct, whose membership and funding is secret. Their goal is to have schools, districts, and states adopt the California standards and they recommend Saxon materials as the answer to today’s problems. They are radicals, out of the mainstream, who use fear to get their way. [See http://www.ed.gov/about/bdscomm/list/mathpanel/4th-meeting/presentations/sherry-fraser.pdf]

Strike 3. In October 2006, two of the California back-to-basics leaders conducted a
A marathon of presentations in Seattle, Washington. Eyewitness reports say that they called for the dismissal of everyone who has been engaged in supporting Washington's math Standards and the WASL (state tests) from the Superintendent of Public Instruction Terry Bergeson on down.

Their other suggestions included replacing the Standards with California’s “world class” standards; purge the state schools of any “reform” curricula; erase the influence of the National Council of Teachers of Mathematics standards; make sure that no decision on math instruction is influenced by any educational research or anyone from a college of education; adopt alternative textbooks, such as those now published in Russia or Singapore; look to mathematicians and "good teachers" while avoiding advice of "mathematics educators" (a rung or two below the night custodian) and teachers whose instruction mirrors constructivist notions, a practice which separates them from the "good teachers."

The back-to-basics incursion was undertaken in support of a new movement in Washington state, alleged to be a grassroots movement, called “Where’s the Math,”

Meanwhile, in California, restrictive criteria on the instructional materials that can be purchased with state funding had eliminated all but a few programs in California’s K-8 classrooms that could be characterized as "reform" programs. Textbook adoption is administered under the Curriculum Commission, which, observers say, has a long history of cronyism, replete with closed-door deliberations and biased commissioners, all of went unchecked. The net effect of these discriminatory practices was to discourage publishers from submitting for review all but one of the K-8 mathematics programs funded by the National Science Foundation.

In the fall of 2006, California’s education policy structure began to weaken. A court order ruled the state textbook adoption process unfair. For the first time in nearly a decade, the California Mathematics Council, an NCTM affiliate, began a watchdog process. Over the summer and fall of 2007, CMC carefully observed K-8 mathematics textbook adoption deliberations and published a blog as commentary. [See adoption blog at www.cmc-math.org]

With the curriculum review process under scrutiny, and the adoption regulations under revision, the Commissioners were more circumspect. Classroom teachers, educators, and publishers were openly critical about the limitations of the state’s “world class” standards, which are largely computational/procedural, making it difficult to create programs which address conceptual learning of mathematics. By December 2007, 40 out of 54 programs submitted were approved. Three of the K-8 curriculum programs on the new adoption list are materials that not only reflect reform principles, but are now officially approved for districts to purchase with state funds because they are aligned with the California mathematics standards (http://www.cde.ca.gov/ci/ma/im/)

Given the policy developments in California and the fact that California math results are lower than corresponding data in Washington [See http://nces.ed.gov/nationsreportcard/states/profile.asp] [See also Notes 2 -4] it is ironic that the Washington “Where’s the Math?" movement has increased its membership, enthusiastically waving the California mathematics standards flag
and denouncing Washington state’s NCTM standards-based reform policy. The group has aggressively pursued and received extensive media coverage, and organized legislative action campaigns.

The website urges its followers to activism:

We must make it clear to the Governor and our elected officials that reformed math is not internationally competitive or recognized for its excellence. Reformed programs such as Everyday Math, TERC/Investigation, Connected Math, CMP, IMP, and other NSF-funded programs are not balanced and do not teach computational fluency which are essential to higher-level math and thinking skills.

“Where’s the Math?” now has branches in major cities around the state, and, amazingly, its members sit on an official advisory panel to the Washington state board of education, along with pro-reform teachers and academics (see www.washmath.com).

Characterizing the “Where’s the Math?” mentality is a video titled “Math Education: An Inconvenient Truth,” the video stars a member of the group who is also a Seattle weather broadcaster, M...J. McDermott.

[See http://www.youtube.com/watch?v=Tr1qee-bTZI,]

The speaker conducts a broadcast-quality lecture in which she extols the virtues of the traditional approach to two-digit multiplication. “If you think that Washington pupils should perform multiplication and division with mastery by the end of fifth grade you must insist that schools and school districts not use TERC’s Investigations in Number, Data and Space, and Everyday Mathematics.” (The texts were shown on camera). (Note: the same TERC program being disparaged has been noted in Holly Concannon’s testimony to the Math Panel as contributing to very successful results by Boston schoolchildren.)

McDermott takes us through the computation exercise, step by step.

\[
\begin{array}{c}
126 \\
x 31 \\
\end{array}
\]

1 \times 6 is 6 and 1 \times 2 is 2

7 8
3 times 6 is 18. Write 8 and we carry the 1. 3 times 2 is 6 plus 1 is 7.

8 0 6
We do the addition. 6. 8 + 2 is 10. We carry the 1. 7+1 is 8,

She shows us chalkboard examples from Investigations and the Everyday Math program approaches and claims that children rarely become efficient, confident, and fluent in computation in the first case. In Everyday Math, she admits that partial products method always works, but she often has trouble remembering which bit gets added to which bits. [See Note 4.]

In what direction are we moving in mathematics education? Should we return to the mentality of The Normal Union Arithmetic, published in 1878?

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3 The authors thank K. Dulle for loaning us his copy of The Normal Union Arithmetic.
No. [See Note 5.]

The way the game is usually played, it’s **three strikes and you’re out.**
1. Multiply 65 by 36.

\[ \text{Operation:} \quad 6 \times 5 = 30; \text{we write the 0 and carry the 3 to the next product:} \quad 6 \times 6 = 36 \text{ and 3 are 39; we write the 3; 3 times 5 are 15; we write the 5 under the 3 and carry the 1 to the next product:} \quad 3 \times 6 = 18, \text{ and 1 are 19; we write the 19: adding, we have} \]

\[ 2340. \]

Note.—Teach the pupil how to do the work first; when he is old enough, show him the reason for the method.

**WRITTEN EXERCISES.**

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**MENTAL EXERCISES.**

1. How many are 10 times 2? 20 times 2? 30 times 3? 40 times 4? 50 times 5?
3. How many words will 12 boys spell, if each boy spells 11 words?
Notes:

1. For further background on Boaler's research, see “Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School” by Boaler and Staples [http://www.tcrecord.org/search.asp?kw=Boaler&x=0&y=0]. See also http://www.sussex.ac.uk/education/profile205572.html.

2. Of the 52 states and other jurisdictions, Washington fourth graders’ NAEP math scores (2005) were lower than those in only 4 jurisdictions. For eighth grade, Washington scores were lower than only 2 jurisdictions nationwide. [See http://nces.ed.gov/nationsreportcard/states/profile.asp]

3. Just after this article was completed, 2007 NAEP data became available. The percentage of eighth grade math students in Washington who performed at or above the NAEP Proficient level was 36%. For California the figure was 24%.[See http://nces.ed.gov/nationsreportcard/states/profile.asp]

4. A parent who viewed the Where’s the Math web site captured things well: “So, because a Seattle weather reporter can't remember which bits to add to which bits, the rest of the country's children may be doomed to view math as a set of arbitrary exercises in rote memorization, rather than as an integrated way of understanding the world?”

5. After reading a draft of the present paper, a friend in Hungary writes about authoritarian approaches to mathematics education: "I'm writing a book, and maybe [will] include some arguments about the responsibility of math teaching in the totalitarian madness of the last century. Math teaching was essential in convincing the majority of people that they were good for nothing but [to] obey."

References

["Some Worry About Potential Bias on the National Math Panel,” EdWeek, May 19, 2006, By Sean Cavanagh]

["White House Suggests Model Used in Reading To Elevate Math Skills," Ed Week, Feb. 15, 2006.]

EPILOGUE

Now that the Panel's report has been issued, some comments:

1. The report can be found at http://www.ed.gov/about/bdscomm/list/mathpanel/index.html.
2. Summaries of the report can be seen at http://www.washingtonpost.com/wp-dyn/content/article/2008/03/13/AR2008031301492.html


3. Reactions to the report can be found at these sites, among others. http://mathpanelwatch.blogspot.com/

http://www.districtadministration.com/pulse/


4. The report can be summarized as “back to basics in elementary school arithmetic (especially fractions) leading to coursework in algebra.”

4.1 Arithmetic. There should be no argument that proficiency in arithmetic should be a major goal of mathematics education at the elementary school level. This is “a good and narrow aim, according to Professor George Polya. [See http://www.mathematicallysane.com/analysis/polya.asp]

Therefore the good and narrow aim of the primary school [is] to teach the arithmetical skills — addition, subtraction, multiplication, division, perhaps a little more. Also to teach fractions, percentages, rates, and perhaps even a little more. Everybody should have an idea how to measure lengths, areas, volumes. If you don’t do that then you are lost in everyday needs of your everyday life. You wish to paint your house. The paint is sold you that is enough, it says on the bottle, that it is enough to paint so and so many square feet. If you don’t know what square feet are then you cannot estimate how many square feet of walls you have. Well, you will not buy the right size of bottle. And that is just a very trivial example. Arithmetical skills, some idea about fractions and percentages, some idea about lengths, areas, volumes, everybody must know this. This is a good and narrow aim of the primary schools, to transmit this knowledge, and we shouldn’t forget it.

But Polya goes on to say that there are higher aims:

However, we have a higher aim. We wish to develop all the resources of the growing child. And the part that mathematics plays is mostly about thinking. Mathematics is a good school of thinking. This was said so many times. The point about it is, “what is thinking?” Well, thinking which you can learn in mathematics is for instance to handle abstractions. Mathematics is about numbers. Numbers are an abstraction. When we solve a practical problem, then from this practical problem we must first make an abstract problem. Mathematics applies directly to abstractions. Some mathematics enables a child, finally should enable a child, at least to handle abstractions. To handle construct formations, to handle abstract structures. Structure is a fashionable
word now. It is not a bad word. I am quite for it.

But I think there is one point which is even more important. Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics.

One can be certain that the arithmetic espoused by Polya is not mindless back-to-basics spouting of arithmetical nonsense-syllables, but a fabric of ideas and actions held together by the sense-making of the pupil.

The fact that a child can say stuff like, “6 X 3 = 18” without hesitation is not a trustworthy indication that he or she knows arithmetic. In research published in Spring 1983 in School Science and Mathematics O’Brien and Casey asked children who had been on a chant-out-their-arithmetic-facts diet, What is 6 X 3? Virtually 100% of the answers were correct.

Then children were asked in individual interviews to give a real-life story or a word problem for 6 X 3 = 18.

These kids had been surrounded by real life for their entire schooling, and for virtually all their school lives they had force-fed arithmetic "facts.”

Incredibly, a large proportion of the children said something like this: "On Monday I bought 6 doughnuts. On Tuesday I bought 3 doughnuts. How many doughnuts did I buy altogether? 18 because 6 X 3 = 18."

Even though answers involving repeated addition (6 doughnuts on Monday, 6 on Tuesday, 6 on Wednesday) were accepted as multiplicative, more than 75% of the responses at grade 4 and 85% of the responses at grade 5 were incorrect. Worse, half the incorrect responses at grades 4 and 5 were stories for 6 plus 3.

So much for the alleged value of the parrot math approach to instruction and the notion that instant production of arithmetic facts (for example, 6 X 3 = 18) is a sufficient condition for thoughtfulness in arithmetic.

What about Polya’s higher aims? What about mathematics beyond arithmetic? Polya’s claim that mathematics is a good school of thinking doesn’t get much play in the Panel's report.

4.2 Algebra. Is it a reasonable guess that the college math teachers on the Panel see lots of weaknesses in fractions and algebra in students’ work in the traditional (outdated?) university courses they are presently teaching? Is the Panel’s emphasis on fractions and algebra equivalent to an elementary school teacher’s saying, “If kids learned to spell—if they’d stop writing gril for GIRL and too for TO and beleive for BELIEVE—they’d have all the essential knowledge to succeed in language and literature/”Is it too optimistic to think that a presidential panel might do more than administer bandaids to a doddering curriculum?

The Panel’s recommendation is that algebra is the be-all and end-all of elementary school mathematics. How about statistics and data-handling and data-representation? Problem solving? Proof? Patterns? [See Annie Selden and Kien Lim, “Developing Mathematical
Habits of Mind,” MAA FOCUS, March 2008.] Given Polya’s call more than 30 years ago for mathematics as a school for thinking, are there not more appropriate goals? Research by Julian Stanley tells us that mathematical talent involves the ability to get to the heart of a situation and to toss away irrelevant attributes. Could this be the heart of a fruitful course for upper elementary and secondary pupils? Is it reasonable that pupils be given a variety of courses to choose from? [See Howard Gardner, “E Pluribus .... A Tale of Three Systems,” in EdWeek, April 23, 2008.]

Based on practices of “high performing” countries, the National Math Panel report recommends schools accelerate mathematics learning, including study of algebra at grade 8. The report offers outlines content of an “authentic” algebra course (the question of what constitutes an inauthentic course will not be discussed here).

A bit of history may provide a useful perspective on the wisdom of this practice. For the past five years, algebra has been mandated for high school graduation in California, and the state’s ten-year-old standards are designed to encourage teaching algebra in 8th grade. Course content is similar to the panel’s syllabus: heavy on symbol manipulation and procedural fluency, light on conceptual understanding. A recent study—the most supportive of the Panel's position we could find— reports that more 8th graders in California schools are now taking algebra; however, the degree of “high performance” appears to be lacking, as only 38% of these 8th grade Algebra I students scored at proficient or advanced levels on the 2006-07 state tests. In the same year, only 17% of those who took Algebra I in 9th grade scored at proficient or advanced. [See “Eighth Graders Score Best on Algebra 1 CST” (February 2008) and “More Students are Taking Algebra 1”(February 2008); See www.edsource.org]

The reports on algebra performance in California are byzantine. State data for 2007 show that only 23 per cent of California pupils were proficient or advanced in Algebra 1. [See http://www.cde.ca.gov/nr/ne/yr07/yr07rel98.asp] For first-time test-takers, the figure was 26%, for repeat test-takers, 15%. And a report by David Foster, presented at the December 2007 meeting of the California Math Council says that (at the end of their junior year) 21% of the graduating class of 2007 met the standard for algebra 1. For algebra 2, the figure was 10%. [David Foster, “The Real Change Agents: Building a Professional Learning Community.” See www.noycefn.org/math/resources]

It is clear that the data do not give much support for the Panel's embrace of California's "world class" curriculum.

5. **Summary.** We should be concerned with children’s construction of thinking. The goal of education should be to enable children (and the adults they will become) to live in equilibrium with their environment and with the demands of new situations, not merely to transmit to children the facts, rules, procedures, conventions, and nomenclature of narrow knowledge. [See T.C. O’Brien, "What's Basic? A Constructivist View," Basic Skills and Choices 1 (Washington, D.C.: National Institute of Education, April 1982), pp. 85-94. Copies available from the author.]

English and Watters say it succinctly:
Children today are facing a world that is shaped by increasingly complex, dynamic, and powerful systems of information in a knowledge-based economy (…). Being able to interpret and work with complex systems involves important mathematical processes that are under-emphasized in numerous mathematics curricula, such as constructing, explaining, justifying, predicting, conjecturing and representing, together with quantifying, coordinating, and organising data. 


In 1977 the mathematician Hans Freudenthal gave his retirement address at the University of Utrecht after a distinguished career as a mathematician and as a contributor to mathematics education. “What have I tried to do with my career?”, he said. I have tried to make important ideas trivial.”

Puzzled as to how important ideas could be trivial, I [O’Brien] asked Freudenthal to elaborate.

I have tried to make important ideas known to everyone. For example, there was once a time when only one man in the history of the human race knew the equation E=mc². As I am sure you know, that was Professor Einstein. Now, not very many years later, even modestly talented secondary students in math and physics all over the world can derive the equation E=mc².

In contrast with Freudenthal’s goal of making important ideas trivial (i.e., widespread) it is a reasonable summary to say that the National Mathematics Advisory Panel has tried to make trivial ideas seem important.