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FOR THE REST OF YOUR LIFE
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‘For the Rest of Your Life’ is a new TV game show. Contestants play to win money every month. This can be for as little as one month or, if every one of their guesses is correct, for the rest of their lives. The rules are shown in table 1.

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Table 1

The first problem that will be analysed is: In the first part of the programme, if the contestant stops as soon as he/she has £600, how likely is it that he/she will win £600?

The scenarios that will win £600 are the tubes being drawn in the following orders.

1) WWWW Probability = $\frac{8}{11} \times \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{7}{33}$

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2) WWWRWW Probability = 4 x (8/11 x 7/10 x 6/9 x 3/8 x 5/7 x 4/6) = 8/33  
3) WWRWWW  
4) WRWWWWW  
5) RWWWWW  
6) WWWRRWWW Probability = 14 x ( 8/11 x 7/10 x 6/9 x 3/8 x 2/7 x 5/6 x 4/5 x 3/4)  
7) WWWWRWWW = 14/55  
8) WWRWWW  
9) WWRWWW  
10) WWRWWW  
11) WRWWW  
12) WRWWW  
13) WRWWW  
14) WRRWWW  
15) RRRWWW  
16) RRRWWW  
17) RRRWWW  
18) RRRWWW  
19) RRRWWW  

Probability £600 is won = 7/33 + 8/33 + 14/55 = 117/165

The contestant has a good chance of winning £600 - approximately ¾
Suppose, however, he/she decides to try for £750.

The second problem that will be analysed is: In the first part of the programme, if the contestant stops as soon as he/she has £750, how likely is it that £750 will be won and is it worth trying for £750?

The scenarios that will win £750 are the tubes being drawn in the following orders.

1) WWWWWW Probability = 8/11 x 7/10 x 6/9 x 5/8 x 4/7 = 4/33  
2) WWWWRWWW Probability = 5 x ( 8/11 x 7/10 x 6/9 x 5/8 x 3/7 x 4/6 x 3/5)  
3) WWRWWWWW = 2/11  
4) WWWWWWW  
5) RWWWWW  
6) RWWWWW  
7) WWWRRRRWWW Probability = 20 x ( 8/11 x 7/10 x 6/9 x 5/8 x 3/7 x 2/6 x 4/5 x ¾ x 2/3)  
8) WWWRRRRWWW = 8/33  
9) WRRRRWWWW  
10) WWWRRRRWWW  
11) WWWRRRRWWW  
12) WRRRRWWWW  
13) WRRRRWWWW  
14) WRRRRWWWW  

Probability of winning £750 = 4/33 + 2/11 + 8/33 = 6/11
Interestingly, the contestant has a reasonable chance of winning £750
If we look at the expected winnings, however, we see that the contestant is better off trying for £600

Expected winnings, given that he/she is trying for £600, = £600 x 117/165 = £425
Expected winnings, given that he/she is trying for £750, = £750 x 6/11 = £409

Let us suppose that the contestant takes £600 into the second part of the programme.
What strategy should the contestant use to maximise their expected winnings?
Suppose the contestant tried to win £600 a month for the rest of his/her life. The probability of pulling out 11 white lights in succession is
11/15 x 10/14 x 9/13 x 8/12 x 7/11 x 6/10 x 5/9 x 4/8 x 3/7 x 2/6 x 1/5 = 11! 4!/15! = 1/1365
clearly it is not in his/her interest to try and win the money for the rest of his/her life!

In fact most contestants adopt a strategy of playing until only one red light remains.
This strategy will be analysed.

**What are the expected winnings of a contestant who plays until three red lights have been revealed?**

(In fact it may be the case that the contestant who adopts this strategy never actually sees three red lights – all the white lights may show before three red lights are revealed.)
The outcomes, their associated probabilities and winnings are shown in table 2.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability (=p)</th>
<th>Winnings (=W)</th>
<th>pW</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 white</td>
<td>1/1365</td>
<td>£600x480</td>
<td>£600x480/1365</td>
</tr>
<tr>
<td>11 white, 1 red</td>
<td>1/1365</td>
<td>£600x60</td>
<td>£600x132/1365</td>
</tr>
<tr>
<td>11 white, 2 red</td>
<td>66/1365</td>
<td>£600x180</td>
<td>£600x180x66/1365</td>
</tr>
<tr>
<td>11 white, 3 red</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10 white, 3 red</td>
<td>132/1365</td>
<td>£600x24</td>
<td>£600x24x180/1365</td>
</tr>
<tr>
<td>9 white, 3 red</td>
<td>165/1365</td>
<td>£600x12</td>
<td>£600x12x180/1365</td>
</tr>
<tr>
<td>8 white, 3 red</td>
<td>180/1365</td>
<td>£600x6</td>
<td>£600x6x168/1365</td>
</tr>
<tr>
<td>7 white, 3 red</td>
<td>180/1365</td>
<td>£600x6</td>
<td>£600x6x168/1365</td>
</tr>
<tr>
<td>5 white, 3 red</td>
<td>147/1365</td>
<td>£600x3</td>
<td>£600x3x147/1365</td>
</tr>
<tr>
<td>4 white, 3 red</td>
<td>120/1365</td>
<td>£600x1</td>
<td>£600x1x120/1365</td>
</tr>
<tr>
<td>3 white, 3 red</td>
<td>90/1365</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 white, 3 red</td>
<td>60/1365</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1 white, 3 red</td>
<td>33/1365</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3 red</td>
<td>12/1365</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2
(Note that it is not possible to pull 11 white lights and 3 red lights using this strategy. Since the last light pulled is red the 11th white light will have been pulled previously. The contestant stops pulling once the 11th white light has been pulled and hence it is not possible to pull 11 white lights and 3 red lights.)

(To see how these probabilities are calculated consider the probability of 5 white and 3 red

\[
\text{5 white} \quad \text{2 red} \quad \text{last one red} \quad \text{number of combinations of 5 white and 2 red}
\]

\[
= \frac{11! \times 4! \times 7! \times 7!}{6! \times 15! \times 5! \times 2!}
\]

(Note that the last tube picked has to be red because the contestant stops pulling once 3 red lights show)

In general the probability of \( X \) white and 3 red is

\[
\frac{11! \times 4! \times (15 - (X+3))! \times (X + 2)!}{15! \times (11 - X)!}
\]

where \( X < 11 \)

The expected winnings are \( \sum pW = £16513. \)
Not bad winnings for pulling lights out of a tube at random!!