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Risk: Mathematical and Otherwise

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Risk: Mathematical *and* Otherwise

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Abstract: What role might mathematicians have to play in the management of risk? The idea of turning a risk, a *possibility* of loss or injury, into a “calculated” risk, a quantified *probability* of loss or injury, is one that has obvious appeal not just to statisticians and mathematicians – but to large numbers of others who would like to know the probability of failure before pursuing some intended course of action. Conclusion: even when risks can be calculated with great precision, they can only be used to inform judgment, but not substitute for it. And it matters *who* is making the judgment.

Keywords: risk compensation, virtual risk, probability, risk amplification.

“It was a calculated risk, and we forgot to carry the one.”

[Thanks to Mark Anderson for permission to reproduce]

In 1999 NASA’s Mars Climate Orbiter burned and crashed because no one had thought to check whether force, expressed in pounds, had been converted to force expressed in Newtons (Grossman, 2010). Failure to carry the one, or convert pounds to Newtons, are examples of only one of the risks encountered in attempting to apply mathematics to the management of risk.

Assuming they can remember to carry the one, what role might mathematicians have to play in the management of risk? The idea of turning a risk, a *possibility* of loss or injury, into a “calculated” risk, a quantified *probability* of loss or injury, is one that has obvious appeal not just to statisticians and mathematicians – but to large numbers of others who would like to know the probability of failure before pursuing some intended course of action.

“Risk” (almost a billion Google hits) has become a booming business. “Risk management” yields over 80 million hits, and “chief risk officer” (of interest to those looking for employment in this field) returns half a million. Governments are keen on risk management: Turnbull, Basel, Sarbanes-Oxley are names associated with guidance, accords or legislative acts intended to ensure that financial risks are managed effectively. Most big banks now have extraordinarily highly paid chief risk officers (CROs) to ensure compliance with their requirements – in 2011 the CRO at Bank of America was paid $11.4 million (Bloomberg News, 2011). Other large, non-financial, enterprises such as General Motors and Ford, Shell and BP, Delta Airlines, Toyota, also have senior executives bearing the CRO title.

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The financial meltdown of 2007/2008 gave a huge boost to the risk management industry. It has now declared itself a profession and it is growing at an impressive rate: GARP, the Global Association of Risk Professionals grew more than three-fold from 55,000 members pre-crash in 2006 to more than 175,000 by 2011.

Types of Risk

The growing army of risk managers seeks to manage an extraordinary range of different risks. Here is a short starter list: financial risk (credit risk, market risk, liquidity risk, value at risk …), legal risk, reputation risk, medical risk, strategic risk, policy risks, inflation risk, recession risk, terrorism risk, sanctions risk, climate risk, radiation risk, extreme weather risk, road accident risk, etc., etc.

The list could go on almost without end. Any threat of nature or any human activity, physical or intellectual, leading to an uncertain outcome can serve as a descriptor of a type of risk.

A further, less open-ended, set of categories can be helpful in an attempt to illuminate the challenges facing risk managers seeking to reduce risks to calculable probabilities. Figure 1 presents a risk typology that is germane to most discussions of a wide variety of risks and their management.

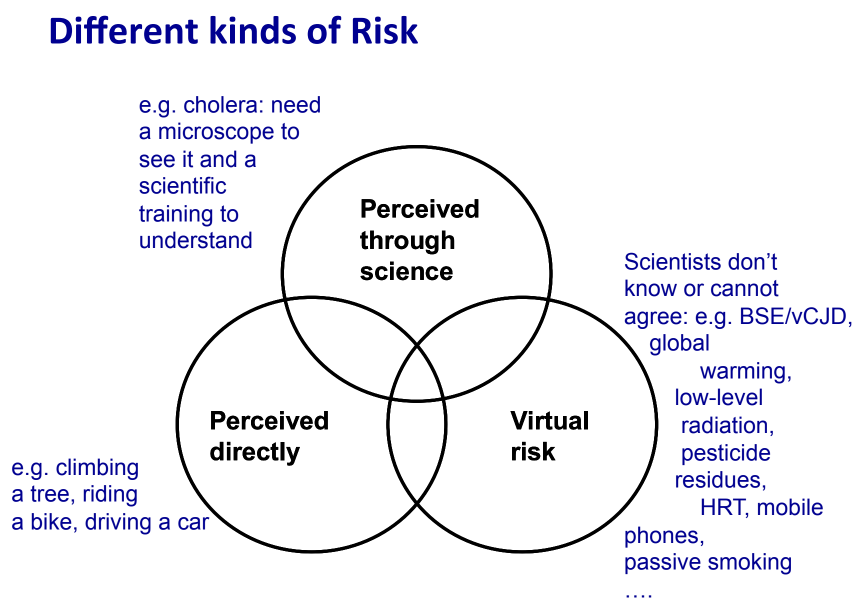

Figure 1. Different kinds of risk

The Venn diagram in Figure 1 suggests that the typology can be useful to distinguish three different, but not mutually exclusive, types of risk. One need sample only a tiny fraction of the 100s of millions of Google "risk" hits to discover unnecessary and often acrimonious arguments caused by people using the same word to refer to different things and shouting past each other. The typology offered in Figure 1 can help to dispose of some unnecessary arguments and, perhaps, civilize others.

Risks in the *perceived directly* circle are managed using *judgement*. We do not undertake a formal, probabilistic risk assessment before crossing the road; some combination of instinct, intuition and experience usually sees us safely to the other side. The consequences of failing to carry the one, or convert pounds to Newtons are, like road accidents, usually the result of carelessness: a failure pay attention to directly perceptible hazards.

The second, the *risk-perceived-through-science* circle, dominates the risk management literature. This is the circle within which most of the risk professionals ply their trade. It is the mathematical circle. In this circle we find books, reports and articles with verifiable numbers, causeand-effect reasoning, probability and inference. This is the domain of, amongst others, biologists with microscopes searching for microbial pathogens and astronomers with telescopes plotting the courses of incoming asteroids. This circle contains contributions from the whole range of science, technology and the social sciences – from physics and chemistry to epidemiology and criminology. But the central science is statistics – the discipline that has probability at its core. The future can be imagined with the help of statisticians, but only *if one is happy to assume* that the historic relationships embodied in their models will persist unchanged into an uncertain future.

The circle labelled *virtual risk* contains contested hypotheses, ignorance, uncertainty and unknown unknowns. If an issue cannot be settled by science and numbers, we rely, as with directly perceptible risks, on *judgement*. Some find this enormously liberating; interested parties are freed to argue from their beliefs, prejudices or superstitions. It is in this circle that we find the longest-running and most acrimonious arguments. Virtual risks may or may not be real, but beliefs about them have real consequences. Global warming has been placed in this circle because the (potentially catastrophic?) warming of which some warn, and which others dispute, is the product of models that grossly simplify extremely complex systems, but lead some to propose policies that would, if pursued, dramatically alter the life-styles of billions.

Risk on the Road: Numbers, and Arguments about Numbers

We can find all three of these risk types contending on the road. In order to contain the discussion in this essay within reasonable bounds I will focus mainly on examples from the realm of road safety. Road safety is an issue that comes with a large number of numbers attached. And they settle few arguments.

People living alongside roads with high volumes of fast traffic often complain, relying on their *direct perceptions*, that their roads are dangerous, and campaign for measures that will reduce the volume and slow the speed of the traffic outside their front doors. Their campaigns sometimes bring them into contact with the highway engineers responsible for their roads. The engineers are likely to confront them with numbers from the *mathematical* circle of Figure 1. Their road accident hot-spot maps show that the roads complained of are safe, with no, or very few, accidents. But the people living alongside the road are unpersuaded by the numbers on the engineers' maps. They can *see* that their roads are dangerous.

The road environment also throws up numerous problems that can be consigned to the "virtual" circle of Figure 1 – issues about which people cannot agree or confess ignorance. At what age is it safe to allow your children to get to school on their own, or cross a busy road? How will driverless cars interact with pedestrians and cyclists? Should cyclists be compelled to wear helmets, or motorists seat belts[1]? All these are current on-going debates that spring to mind.

Crossing the road

I offer Figure 2 as a simple model of what goes on in my head when I am crossing the road. I call it the risk thermostat. The thermostat is set in the top left-hand corner. The setting of risk thermostats can vary enormously – from that of a timid and cautious little old lady named Prudence to that of a wild and reckless Hell's Angel. But everyone has some propensity to take risks; a zero risk life is not possible.

[1] Readers who thought that this debate had been settled are referred to "Britain's seat belt law should be repealed" (http://john-adams.co.uk/wp-content/uploads/2008/08/seat-belts-for-significance-2.pdf). It is a debate that goes back to 1982 (see, for example, http://john-adams.co.uk/wp-content/uploads/2006/SAE%20seatbelts.pdf) and more recently (see, for example, http://www.john-adams.co.uk/?s=seat+belts)

A *propensity* to take risks leads to risk taking behaviour that leads, by definition, to *accidents.* To take a risk is to do something that carries with it a probability of an adverse outcome. Through having accidents, and surviving them and learning from them, or seeing them on television, or being warned long ago by mother, I have acquired a *perception* of the risks associated with crossing roads. The model proposes that when my perception of a risk and my propensity to take it are out of balance I change my behaviour to restore the balance. Why do I cross the road? To get to the *reward* on other side; and the magnitude of that reward will influence the setting of my thermostat. The change in behaviour in response to changes in the perception of risks described by Figure 2 is commonly known as *risk compensation.*

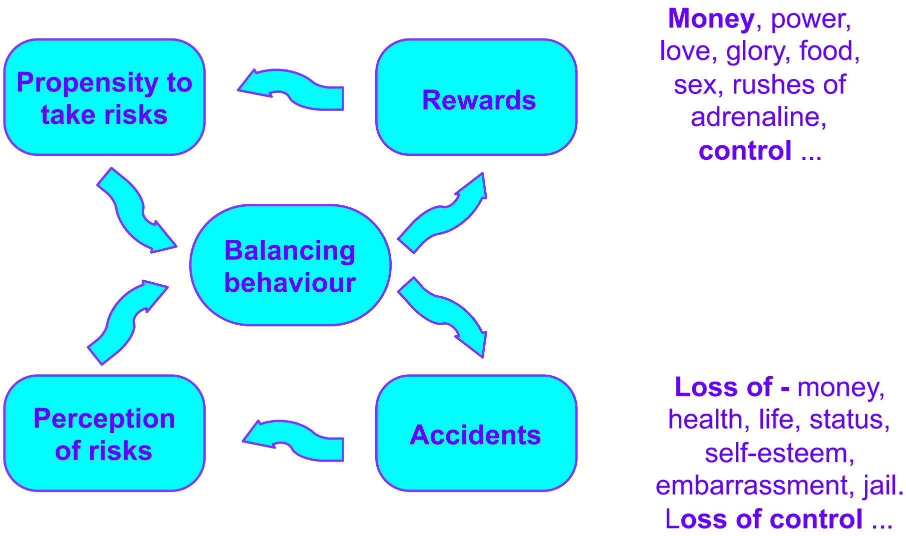

Figure 2. The risk thermostat

I used to describe the process illustrated by Figure 2 as cost-benefit analysis without the £ or $ signs, but nothing, it appears, is beyond the determined economist's ability to be rendered as a sum of money. Spending on drugs and other medical procedures is commonly justified in terms of the Qalys (quality adjusted life years) that they would yield – the UK value of a Qaly currently ranges from £10,000 to £70,000 (Donaldson et al., 2011). A significant benefit claimed for new road schemes in Britain is the value of the lives that they would save – with each life currently valued at £1,249,890 (2006 value, routinely adjusted for inflation: http://webarchive.nationalarchives.gov.uk/20100304070241/http://www.dft.gov.uk/pgr/economics/software/coba11usermanual/part2thevalofcostsandb3154.pdf). And the Stern Report, an influential contribution to the climate change debate in Britain has sparked a debate framed in terms of the monetary costs and benefits, and their discount rates, likely to be incurred or enjoyed many generations into the future (see, for example, http://en.wikipedia.org/wiki/Stern_Review#The_costs_of_mitigation).

What kills you matters

But it appears that in the eyes of many non-economists, some pounds or dollars are more equal than others. In listing some of the contents of the *Rewards* and *Accidents* boxes in Figure 2 *control* and *loss of control* have been highlighted. Figure 3 sets out the significance of this factor.

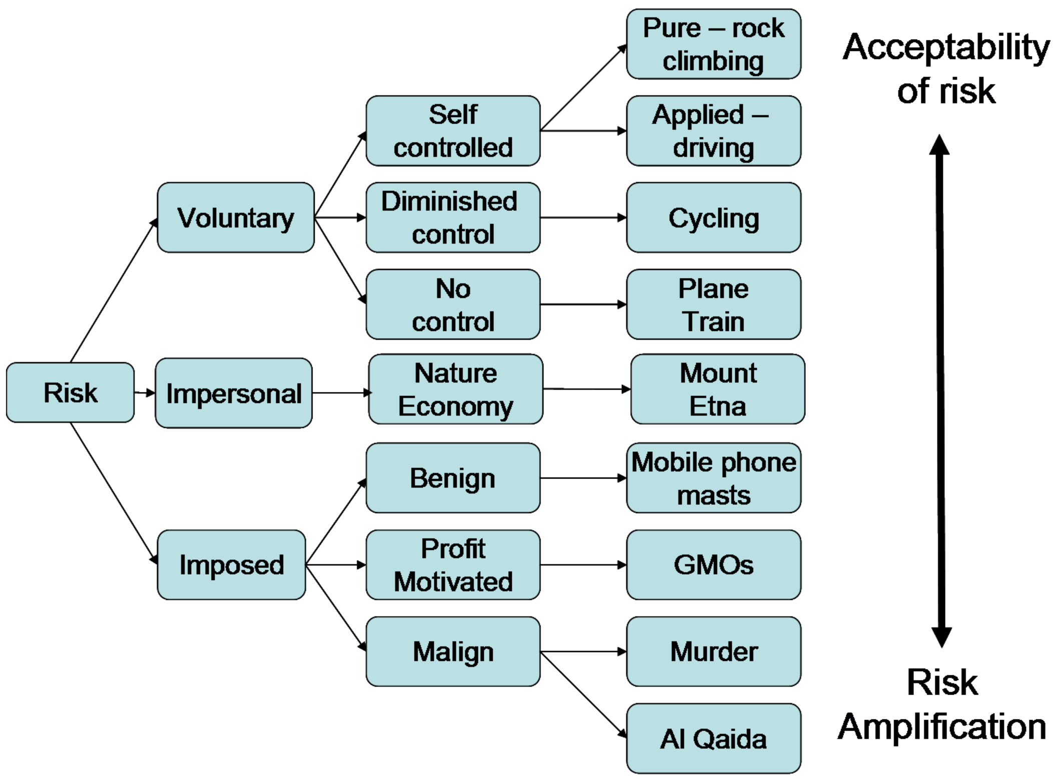

Figure 3. What kills you matters

Acceptance of a given actuarial level of risk varies widely with the perceived level of control an individual can exercise over it and, in the case of imposed risks, with the perceived motives of the imposer.

With 'pure' voluntary risks, the risk itself, with its associated challenge and rush of adrenaline, is the reward. Most climbers on Mount Everest and K2 know that it is dangerous and willingly take the risk. Similarly thrill-seeking young men driving recklessly are aware that what they are doing is dangerous; that is the point.

With a voluntary, self-controlled, applied risk, such as driving, the reward is getting expeditiously from A to B. But the sense of control that drivers have over their fates appears to encourage a high level of tolerance of the risks involved.

Cycling from A to B (I write as a London cyclist) is done with a diminished sense of control over one's fate. This sense is supported by statistics that show that per kilometre travelled a cyclist is much more likely to die than someone in a car. This is a good example of the importance of distinguishing between relative and absolute risk. Although much greater, the absolute risk of cycling is still small – 1 fatality in 25 million kilometres cycled; not even Lance Armstrong can begin to cover that distance in a lifetime of cycling. And numerous studies have demonstrated that the extra relative risk is more than offset by the health benefits of regular cycling; regular cyclists live longer.

While people may voluntarily board planes, buses and trains, the popular reaction to crashes in which passengers are passive victims, suggests that the public demand a higher standard of safety in circumstances in which people voluntarily hand over control of their safety to pilots, or bus or train drivers.

Risks imposed by nature – such as those endured by people living on the San Andreas Fault or the slopes of Mount Etna – or by impersonal economic forces – such as the vicissitudes of the global economy – are placed in the middle of the scale. Reactions vary widely. Such risks are usually seen as motiveless and are responded to fatalistically – unless or until the risk can be connected to base human motives. The damage caused by Hurricane Katrina to New Orleans is now attributed more to wilful bureaucratic neglect than to nature. And the search for the causes of the economic devastation attributed to the 'credit crunch' is now focusing on the enormous bonuses paid to the bankers who profited from the subprime debacle.

Risks imposed by one's fellow humans are less tolerated. Consider mobile phones. The risk associated with the handsets is either non-existent or very small. The risk associated with the base stations, measured by radiation dose, unless one is up the mast with an ear to the transmitter, is orders of magnitude less. Yet all around the world billions of people are queuing up to take the voluntary risk, and almost all the opposition is focused on the base stations, which are seen by objectors as impositions. Because the radiation dose received from the handset increases with distance from the base station, to the extent that campaigns against the base stations are successful, they will increase the distance from the base station to the average handset, and thus the radiation dose. The base station risk, if it exists, might be labelled a benignly imposed risk; no one supposes that the phone company wishes to harm all those in the neighbourhood. And the extent to which traffic is seen as an imposed risk varies widely; parents of young children and cyclists are much more likely to feel it as an imposition than drivers of SUVs and big cars.

Even less tolerated are risks whose imposers are perceived to be motivated by profit or greed. In Europe, big biotech companies such as Monsanto are routinely denounced by environmentalist opponents for being more concerned with profit than the welfare of the environment or the consumers of its products. Manufacturers of high-performance cars are assigned by some road-safety campaigners to the same category, their arguments sometimes adding damage to the environment to the danger posed to vulnerable road users.

Less tolerated still are malignly imposed risks – crimes ranging from mugging to rape and murder. In most countries the number of deaths on the road far exceeds the numbers of murders, but far more people are sent to jail for murder than for causing death by dangerous driving. In the United States in 2012 14,827 people were murdered – a statistic that evoked far more popular concern than the 33,561 killed on the road – but far less concern than that inspired by the zero killed by terrorists.

Which brings us to Al Qaida, Isis and their associates. How do we account for the massive scale, world-wide, of the outpourings of grief and anger attaching to its victims, whose numbers are dwarfed by victims of other causes of violent death? In London 52 people were killed by terrorist bombs on 7 July 2005, about six days worth of death on the road in the whole country. But thousands of people do not gather in London's Trafalgar Square every Sunday to mark, with a three-minute silence, their grief for the previous week's road accident victims.

The dangers that can be tracked to the malign intent of terrorists are amplified by governments who see them as a threat to their ability to govern – to their ability to control events. To justify forms of surveillance and restrictions on liberty previously associated with tyrannies, 'democratic' governments now characterize any risk to life posed by terrorists as a threat to *Our Way of Life*.

Moving from the top to bottom of Figure 3 we encounter a phenomenon known as risk amplification. *The numbers almost don't matter.* Figures 2 and 3 can also help to explain the discrepancy referred to above between the judgments of local residents and the mathematically based judgement of the highway engineer about the safety of a road. If the residents perceive their road to be dangerous they will modify their behaviour. Old people will be afraid to cross it. Children will be forbidden to cross it. And fit adults will cross it quickly and carefully. Their good accident record is often purchased at the cost of community severance. People on one side of the road tend no longer to know their neighbours on the other side. The numbers on the highway engineer's accident map measure not safety, but risk aversion. And those living on the road will tend to see the danger as an imposed risk, amplifying their perception of the risk.

Some more numbers from Great Britain

Staying with risk on the road, Figure 4 describes the rise and fall of road accident deaths in Great Britain between 1950 and 2012. I use Great Britain as an example because it is the country with which I am most familiar, but most highly motorised countries display similar patterns over this period. GB road accident fatalities reached a post-war peak of 7985 in 1966 before falling to 1713 in 2013 – the lowest since records began.

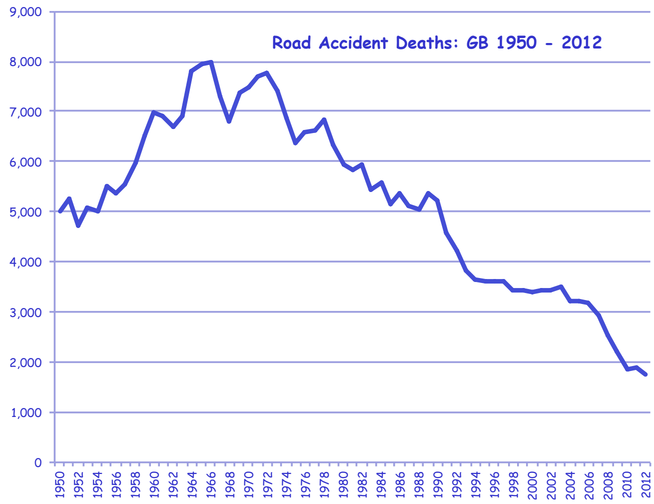

Figure 4. Road accident deaths: Great Britain (1950-2012)

How might the numbers represented by this graph be explained? Over the period traffic grew considerably; cars improved in terms of crash protection and better brakes; numerous laws were passed to curb speed, compel the use of seat belts and helmets, and ban drinking and driving and the use of mobile phones. And highway engineers lengthened sight-lines, installed central barriers on freeways and pedestrian barriers in cities, and removed roadside obstacles such as trees.

So, was it the engineers with their improved brakes and crash protection? Was it the road builders with their safer roads? Was it the legislators and legislation enforcers? Who deserves the credit for this extraordinary reduction in road accident fatalities?

Figure 4 transforms Figure 3 in a way that sheds some light on the possible risk reduction effect of all these measures. It represents fatalities per unit of exposure – i.e. per billion vehicle kilometres of traffic. It shows a 96% decrease over the 62-year period displayed on the graph,

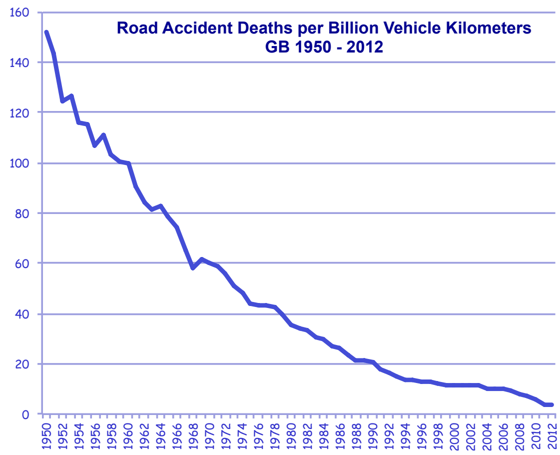

Figure 5. Road accident deaths per billion vehicle kilometers : Great Britain (1950-2012)

And Figure 5, with the vertical axis logged, transforms the graph again in a way that poses challenging questions to those who would claim credit for any of the risk reduction measures listed above.

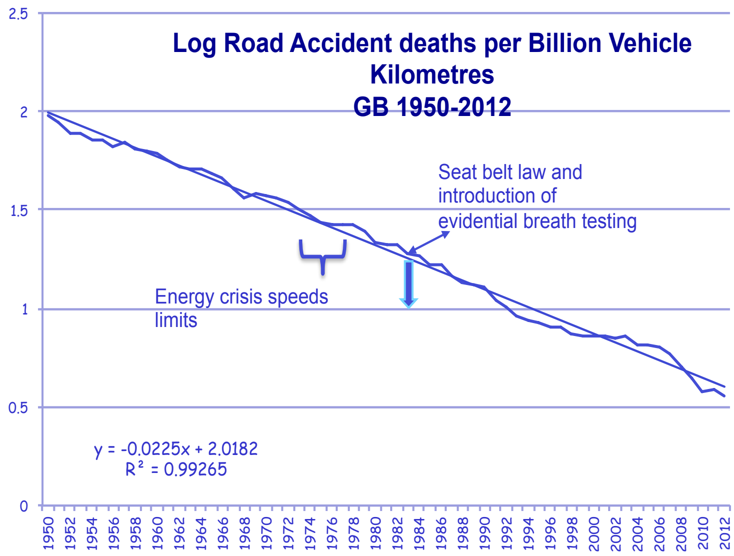

Figure 6. Log road accident deaths per billion vehicle kilometres: Great Britain (1950-2012)

The slope of the straight line indicates that, over this period, fatalities per kilometre fell by an average of 5.3% per year. Over most of the period before 1966 traffic increased faster that 5.3% per year and afterwards more slowly with the result that over the whole of the period fatalities per kilometre declined.

But it is extraordinarily difficult to spot the contributions of the vehicle engineers and manufacturers, the legislators and the road builders referred to above. Over this period the interventions whose promoters promised would have the largest and most immediate effects were the energy crisis speed limits and, in 1983, the seat belt law and the introduction, in the same year, of evidential breath-testing. Both promised instant large downward steps on the graph in Figures 5 and 6 and both are very difficult to see. The largest single step down over the whole period was in 1991 when nothing significant happened on the road safety front – except for the most severe economic recession since the war.

In Britain, over this period, the largest road safety claims, by a wide margin, have been made by the seat-belt campaigners. In 2008, the 25th anniversary of the seat-belt law, the Department for Transport, the Parliamentary Advisory Council on Transport Safety and the Royal Society for the Prevention of Accidents all published press releases claiming credit for their contribution to the creation of a law that had saved 60,000 lives over the previous 25 years.

Here we have an opportunity for mathematicians

The claims are outrageous nonsense. British mathematical enthusiasts were asleep at the switch. The downward arrow on Figure 6 illustrates the magnitude of the sharp downward step that should have occurred on the graph if the claims (averaging 2400 lives a year over 25 years) were true.

Legislators and engineers have, for many years, routinely over-claimed for their safety achievements (Adams, 1985). The problem is not confined to the road safety arena. Figure 7 by Leeth and Hale (2013) in their examination of the effect of the Occupational Safety and Health Act of 1970 suggests little has changed over the intervening decades. On their graph, displaying a downward trend similar to that in Figure 6, it is very difficult to discern the much-heralded effects of the Act.

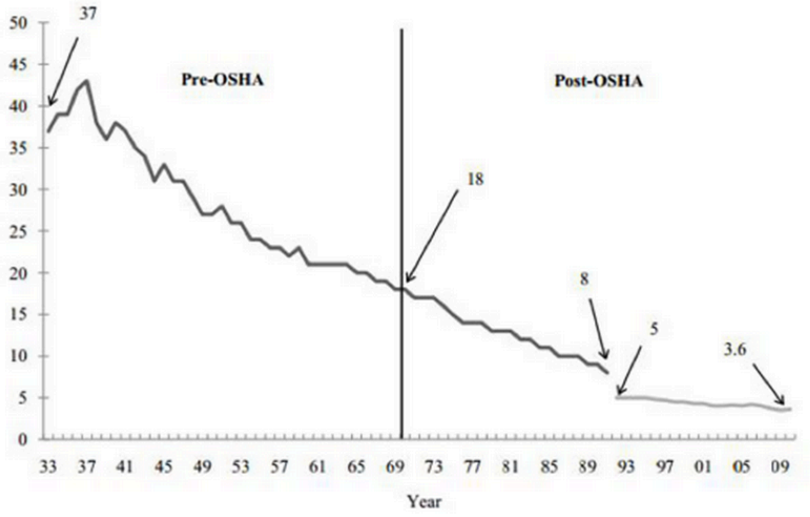

Figure 7. An evaluation OSHA's effectiveness
[Reproduced with the kind permission of the authors]

Almost 35 years ago Laurence Ross (1976), in the Scandinavian Myth, used a similar line-on-a-graph method to challenge the widely proclaimed view that Sweden, with its low permitted alcohol levels, strict enforcement, and draconian punishments was a model for the rest of the world to follow. His interrupted time-series analyses revealed no effect of the Scandinavian drink-drive laws on the relevant accident statistics.

His analysis suggested that tough drink-drive legislation is only likely to work where it accords with prevailing public opinion. He noted the existence of a politically powerful temperance tradition in Scandinavia. Many people considered drinking and driving a serious offence (if not a sin) before it was officially designated as such by legislators. The absence of a detectable effect of Scandinavian drink-drive laws on accident and fatality statistics at the time the laws came into effect suggested, according to Ross, that the laws were symptomatic of a widespread concern about the problem, and that most people likely to obey such laws were already obeying them before they were passed. The laws, in effect, simply ratified established public opinion.

I offer an examination of safety claims as a challenge to mathematical enthusiasts everywhere. It is a game the whole class can play. And it can be more than a game. It can be an introduction to the fascinating world of risk. There are vast numbers of risk management proposals and claims begging to be tested by mathematical enthusiasts. The risk-management "starter list" presented at the beginning of this essay merely scratches the surface.

In conclusion I return to risk on the road and invite others to share my fascination with the problems it presents. This is where I began over 40 years ago when challenging the Government's road safety arguments at public inquiries and I still find some of the questions it raises challenging.

A final set of numbers – and a hypothesis

How might we account for the dramatic fall in numbers of those killed on the road as traffic increased since the Second World War in economically developed countries such as Britain? I offer Figure 8 as a basis for a hypothesis.

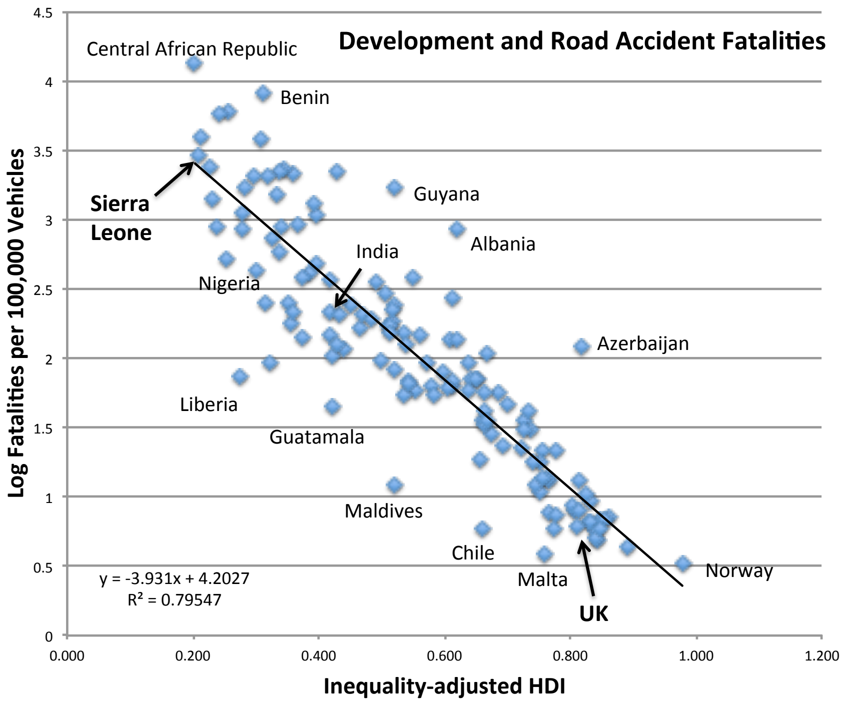

Figure 8. Development and road accident fatalities

The Central African Republic at the top of the top of the graph has a fatality rate per vehicle more than 3000 times higher than Norway at the bottom. And yet it has, along with most of the other countries at the top end, a full set of road safety laws: national speed limits, drink-drive limits, helmet laws, seat belt laws, child restraint laws and laws forbidding the use of mobile phones while driving. And they are all achieving their extraordinary kill rates per vehicle with modern imported vehicles with a hundred years of safety technology built into them. Norway's superior roads also appear unlikely to contribute to the difference; it is often remarked that potholes are nature's speed humps.

It appears that the process of "development" is accompanied by increased risk aversion and a growing sense of collective responsibility. A strong correlation exists between a country's score on the Equality Adjusted Human Development Index and its road death rate per vehicle. Created by Mahbub-ul-Haq and Nobel Laureate Amartya Sen, the Inequality-Adjusted Human Development index is a composite of average longevity, education and income, adjusted for inequality (http://en.wikipedia.org/wiki/Human_Development_Index). The largest outliers have been identified as a spur to further research.

For people living through the period represented by Figures 5 and 6 it would be difficult to perceive their roads getting 5.3% safer year on year. But that 62-year period has witnessed extraordinary societal change, and not just in the workplace as noted in Figure 7. As a child I can remember my respectable parents urging "one for the road" on departing guests. Now drunken driving has become a stigmatizing offence.

Over this period the freedom of children has been severely constrained. I grew up as a free-range child at liberty to roam the neighbourhood until the streetlights came on and expected to get to school on my own. A study of English schools in 1971 revealed that 80% of 7 and 8 year old children got to school on their own, unaccompanied by an adult. A follow-up study of the same schools in 1990 revealed that that number had fallen to 9% - and the main reason parents gave for denying their children the freedom that they had enjoyed as children was fear of traffic (Hillman, Adams & Whitelegg, 1990). And now it has become a legal child-protection issue. In England two controversies recently appeared in the press in which parents were threatened with child protection orders for allowing their children what used to be the widely accepted freedom to get to school unaccompanied (Moore & Maxted, 2010; BBC News, 2010).

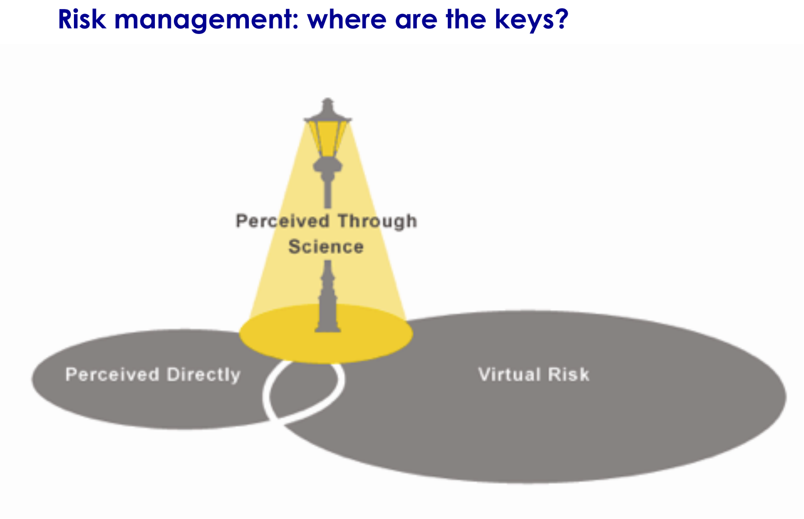

Figure 9. Risk management: where are the keys?

Figure 9 is a redrawn version of the Venn diagram of Figure 1, revised in homage to the mythical drunk who lost his keys in the dark and searched for them under the lamppost because that was where there was light to see. Risk managers searching for the keys to their problems amidst the brightly lit numbers in the mathematical circle are likely to be equally disappointed.

My wife and I usually buy a Euro Lottery ticket every week. The chance of winning is one in 116,531,800. My statistically minded friends jeer at me. My question to them is where else can we buy a week's worth of so much fantasy for £2? We spend an enjoyable week being wonderfully charitable to worthy causes and generous to friends and family – and enjoying a bit of self-indulgence. Our fantasies are highly improbable – but not impossible[2].

Even when risks can be calculated with great numerical precision the numbers can only be used to inform judgment, not substitute for it. And it matters greatly who is making the judgment.

References

Adams, J. (1985). *Risk and Regulation: the record of road safety regulation*. Retrieved from: http://john-adams.co.uk/wp-content/uploads/2007/10/risk%20and%20freedom.pdf

BBC News (2010, September 13). Lincolnshire family warned over girl's bu stop walk. Retrieved from: http://www.bbc.co.uk/news/uk-england-lincolnshire-11288967

Bloomberg News (2011). Risk officer rises to $10 million job after market meltdown. Retrieved from: http://www.bloomberg.com/news/2011-07-11/risk-officer-rises-to-10-million-job-after-derivatives-meltdown.html

Donaldson, C., Baker, R., Mason, H., Jones-Lee, M. W., Lancsar, E., Wildman, J., Bateman, I., Loomes, G., Robinson, A., Sugden, R. C., Prades, J. L. P., Ryan, M., Shackley, P. and Smith, R. (2011). The social value of a QALY : raising the bar or barring the raise? *BMC Health Services Research, 11*(8), 1-8.

[2] This example can serve as an illustration of the, often crucial, distinction between relative and absolute risk. In one recent draw the jackpot was £128 million. My wife bought two tickets. I stuck with one. Her "risk" of winning was twice that of mine [relative risk]. Her risk of winning was 1 in 58,265,900 times greater than mine [absolute risk].

Grossman, L. (2010). Metric math mistake muffed mars meteorology mission. Retrieved from: http://www.wired.com/2010/11/1110mars-climate-observer-report/

Hillman, M., Adams, J., & Whitelegg, J. (1990). *One false move...* Policy Studies Institute: London, England,

Leeth, J. & Hale, N. (2013). *Evaluating OSHA's effectiveness and suggestions for reform*. Retrieved from: http://mercatus.org/publication/evaluating-oshas-effectiveness-and-suggestions-reform

Moore, A. & Maxted, A. (2010). Should the Schonrock children be allowed to cycle to school alone. Retrieved from: http://www.telegraph.co.uk/women/mother-tongue/7872970/Should-the-Schonrock-children-be-allowed-to-cycle-to-school-alone.html

Ross H. L. (1976). The Scandinavian myth: the effectiveness of drinking and driving legislation in Sweden and Norway. *Evaluation Studies – Review Annual vol.*1, Sage.