Influence of site quality on the height-diameter relationship of western larch

Elizabeth D. Reinhardt

Let us know how access to this document benefits you.
Follow this and additional works at: https://scholarworks.umt.edu/etd

Recommended Citation
https://scholarworks.umt.edu/etd/2695
COPYRIGHT ACT OF 1976

This is an unpublished manuscript in which copyright subsists. Any further reprinting of its contents must be approved by the author.

Mansfield Library
University of Montana
Date: 1982
THE INFLUENCE OF SITE QUALITY ON THE HEIGHT-DIAMETER RELATIONSHIP OF WESTERN LARCH

By

Elizabeth D. Reinhardt
A.B., Harvard University, 1978

Presented in partial fulfillment of the requirements for the degree of

Master of Science

UNIVERSITY OF MONTANA

1982

Approved by:

[Signatures]

Chair, Board of Examiners

Dean, Graduate School

8-3-82

Date
The purpose of this study was to explore the relationship between height, diameter, and site quality for western larch (*Larix occidentalis*, Nutt.), and in particular to determine whether height-diameter curves could be used to predict site index. Data included tree height and diameter and an estimate of site index for each of 1369 larch trees. The data were grouped into five site classes, and a height-diameter curve fitted to each class, using least squares regression and testing a number of possible height-diameter equations. Results showed that although height-diameter curves for smaller trees were not differentiated by site, trees larger than 20 inches in diameter reached greater heights at a given diameter on a good site than on a poor site. Height-diameter curves have potential as site index predictors for old growth stands.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>A Review of Site Evaluations</td>
<td>4</td>
</tr>
<tr>
<td>Possible Equations to Describe the Height-Diameter Relationship</td>
<td>12</td>
</tr>
<tr>
<td>Methods</td>
<td>17</td>
</tr>
<tr>
<td>Results</td>
<td>19</td>
</tr>
<tr>
<td>Linear Models</td>
<td>19</td>
</tr>
<tr>
<td>Nonlinear Models</td>
<td>23</td>
</tr>
<tr>
<td>Summary</td>
<td>32</td>
</tr>
<tr>
<td>Discussion</td>
<td>34</td>
</tr>
<tr>
<td>Conclusion</td>
<td>36</td>
</tr>
<tr>
<td>Literature Cited</td>
<td>37</td>
</tr>
</tbody>
</table>
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>Estimated maximum heights and diameters at which maximum heights are reached</td>
<td>20</td>
</tr>
<tr>
<td>Table 2</td>
<td>Estimates of the parameters for the monomolecular equation and the standard deviations of the estimates</td>
<td>24</td>
</tr>
<tr>
<td>Table 3</td>
<td>Comparison of the standard errors of regression and estimates of maximum height obtained from the two forms of the monomolecular equation</td>
<td>26</td>
</tr>
<tr>
<td>Table 4</td>
<td>Parameter estimates and standard deviations for the three parameter Richard's model</td>
<td>28</td>
</tr>
<tr>
<td>Table 5</td>
<td>Standard errors of regression of the two parameter and three parameter Richard's function</td>
<td>28</td>
</tr>
<tr>
<td>Table 6</td>
<td>Parameter estimates of the two parameter Richard's model</td>
<td>29</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>&quot;Segmented&quot; regression lines of log H on log D by site</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>Height-diameter curves by site obtained by fitting the two forms of the monomolecular equation</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>Height-diameter curves produced using the Richard's function with two parameters</td>
<td>30</td>
</tr>
</tbody>
</table>
INTRODUCTION

The purpose of this study was to investigate the relationship between height, diameter and site quality for western larch (Larix occidentalis Nutt). Height-diameter curves were developed and evaluated as predictors of site index.

Site index, the height of dominant trees at a base age of 50 or 100 years, is the conventional descriptor of the potential productivity of a site. The taller a tree is at a given age, the better the site is considered to be. Since height and volume are positively correlated, the site index reflects not just height growth of a tree but, indirectly, potential volume growth of a stand. Possible sources of error in estimating site index include errors in age determination (Husch, 1956; Ferree and others, 1958; Mader, 1963) and errors due to unrepresentative height growth. Height growth may be reduced in very dense or very open stands, and the pattern of height growth within a stand may vary if the trees are not all the same age (Carmean, 1975; McQuilken, 1975). Site index in general is limited to pure, even-aged stands, neither too open nor too dense, in which the dominants and codominants have never been suppressed (Gevorkiantz, 1957).

Height-diameter curves have been proposed as an alternative site index (Meyer, 1940; McLintock and Bickford, 1957; Stout and Shumway, 1982). Diameter can be measured more quickly and easily than age. A height-diameter predictor could save time and reduce the error in site
index estimation caused by errors in age determination. It may also reduce errors caused by suppressed juvenile growth or reduced height growth in an over stocked stand. The height-diameter relationship has been proposed as a site index in stands of uneven age or mixed species.

A site index estimator that could be applied to aerial photographs would be useful for inventory of roadless areas. Choate (1961) studied the relationship of site quality to such factors as slope, aspect, position on slope, slope configuration and latitude, in order to estimate site index from aerial photographs. His results were not completely successful. A height-diameter site quality estimator has potential applications to inventory using aerial photographs, since tree height can be measured on aerial photographs, and tree diameter can be estimated from crown width measurements.

As forest managers begin to practice more intensive management, assessment of site quality becomes increasingly important. Management decisions must be based on estimates of future yield; the accuracy of these estimates depends in part on correct evaluation of site quality. Quantification of site quality must ultimately reflect potential volume yield (Bates, 1918; Mader, 1963; Sammi, 1963). Since volume is a function of both height and diameter, an evaluation of site in terms of height and diameter seems valuable.

Perhaps more importantly, investigation of the relationship between height, diameter and site can contribute to the understanding of the dynamics of forest tree growth. Growth models, such as the Stage Prognosis Model (Wykoff and others, 1981) and the Coniferous Biome
Succession Simulator (Reed and Clark, 1979) often use a single height-diameter equation for growth prediction. These models might be made more sensitive and accurate with a height-diameter equation that includes the influence of site.
A REVIEW OF SITE EVALUATIONS

Forest land is delineated by site quality in order to predict future yield and to put a value on the land. In order to fulfill these functions, site quality classes must reflect the potential of the site to produce wood, but be independent of the history of the stand now occupying the site. Attempts to classify forest sites fall into three categories. The first is environmental or ecological in nature and considers the site itself, the second considers the species found on the site, and the third is based on the growth of the stand supported by the site.

Environmental or ecological classifications are in a sense primary. They look at the factors that influence the quality of a site; the soil, climate or topography. Some of the earliest site classifications were of this kind. Aristotle's successor, Theophrastus (1916), differentiated between forests growing on north-facing and south-facing slopes. In 1872 Blomquist divided Finnish forests into three latitudinal regions, and then divided each region into three quality classes based on soil and vegetation (Cajander, 1926).

This kind of classification has never been developed extensively in this country, although considerable effort has gone into attempts to correlate soil characteristics with site productivity. In 1935 Coile developed a texture-depth site quality index for this purpose. It consisted of the ratio between the silt and clay content of the B₁ soil
horizon and the depth of that horizon below the soil surface. Other studies relate tree growth to soil drainage, organic matter, nitrogen content, gravel content, and depth (Husch and Lyford, 1956; Mader and Owen, 1961; Steinbrenner, 1965). Farrar (1962) developed a site classification system which included eleven moisture regimes, eleven classes of soil pore patterns, three temperature classes and three exposure classes. In general however, it was felt that a simple and accurate classification could not be developed along these lines.

Some recent ecological growth modeling work has renewed interest in environmental indices. Ecological gradients have been developed and tree growth response along these gradients measured. Emmingham and Waring (1977) predicted net photosynthesis, including in their model the effect of temperature, light, and moisture, all of which can be measured or simulated for a particular site. Sites can then be compared in a very fundamental way by estimating a tree's most important response to its environment: net photosynthetic capacity. The Coniferous Biome forest growth model, Succession Simulator (Reed and Clark, 1979) is based on environmental indices, as is the Hubbard Brook growth model Jabowa (Botkin et al., 1972). One of these indices is end of growing season predawn plant moisture stress, a measure which integrates soil moisture stress, atmospheric moisture stress, and the plant's ability to retain water (Waring and Cleary, 1967). Another is optimum temperature days, an index which takes into account daily air and soil temperatures throughout the growing season (Cleary and Waring, 1969). These indices represent an attempt to model the operational environment of the plant,
and seem to be very successful. They require a great deal of data, however, and are not yet adapted to inventory work.

Jones (1969) in his "Review and Comparison of Site Evaluation Methods," includes a review of vegetative type site classifications. Site types based on climax understory vegetation were developed for forests in Finland (Cajander, 1926) and yield tables developed for each type. Most vegetative classifications in the United States have been descriptive in nature and not related to productivity. Examples are Braun's work in eastern deciduous forests (1950) and Daubenmire's work in Idaho (1952). Recently habitat types which include estimates of productivity have been developed for Montana forests (Pfister et al., 1977).

The prevailing trend in the United States has been toward a tree growth approach to forest site classification. Foresters felt that the quality of a site was reflected in the attributes of the stand supported by the site.

For some time there was disagreement over whether volume growth or height growth would be a better basis for site classification. In his 1906 Forest Mensuration text Graves wrote that "experience has shown that under ordinary circumstances the height growth of trees is an excellent index of the quality of a site" although he noted that it was less useful for mixed stands, stands that had been overcrowded in youth, or very open stands. In 1916 Roth proposed a site index based on height and age. He wrote that tree height is sensitive to site, independent of stand density and species mixture, and easy to measure. His thesis has remained in general acceptance ever since, although it was rebutted by Bates (1918)
who wrote that "the only final criterion of site quality is the annual cubic foot increment of a fully stocked stand." In 1917 Watson noted that volume yield, used in Europe at that time to classify sites, was not appropriate for wild stands which were often understocked, of mixed species composition, or uneven-aged. A Society of American Foresters Committee reported in 1923 that height growth was probably a more convenient measure of site quality than ground cover vegetation, soil, climate, species composition of the forest, volume growth, tree form, or color and development of foliage.

The first site index curves were produced graphically. Tree heights and ages were measured on a large number of sample plots, and the data were plotted with age on the x-axis and height on the y-axis. Curves were drawn through the upper and lower ranges of the data, and the interior portion was divided by a series of similar curves (Sterrett, 1921). The curves were used by measuring tree height and age, and reading site index from the graph. The height of the curve at a specified base age, usually 50 or 100, was the site index. One weakness of this method of site index construction is that the shape of the curves was determined by the extremes, the very places where there were least data points (Spurr, 1952). The method was modified so that one hand fitted curve was drawn through the middle of the data, and this curve was used to produce a series of harmonized curves (Bruce, 1923, 1926). The harmonized curves were constructed so that the height of each curve was a given proportion of the guide curve's height at every age. Harmonized curves are based on the assumption that height growth follows the same pattern on
all sites. In 1931 Bull followed actual height growth of red pine on various sites over a period of time and concluded that anamorphic curves were not adequate, as trees growing on good sites had a notably different pattern of growth than those on poor sites. Polymorphic, or natural site curves, as they are sometimes called, can be constructed either by using stem analysis to follow the past growth of a tree, or with permanent growth plots. Each curve is independently produced, and the curves may have different shapes. Osborne and Schumacher (1935) proposed an empirical method of site curve construction which did not require stem analysis or permanent growth plots to produce polymorphic curves. Heights and ages were measured on sample plots, and regression was used to fit one curve to all the data. Then the data were grouped by age interval, and the coefficient of variation was computed for height in each interval. If the coefficient of variation was fairly uniform across the range of ages, anamorphic curves were used. If not, the curves were spaced at each age according to the magnitude of the coefficient of variation at that age.

Recently there has been interest in developing polymorphic curves mathematically. Height is regressed on age and site index, so that a set of curves can be produced from one equation. The equation is chosen so that polymorphic curves result (Brickell, 1968; Lundgen and Dolid, 1970; Ek, 1971; Graney and Burkhart, 1973; Payendeh, 1974; Monserud, 1978; Carmean and Hahn, 1981).

Height-age site curves have not escaped criticism. In 1956 Spurr reported that height growth patterns may vary even on sites of the same
quality. Trees growing on shallow soil may show normal early growth, but reduced growth later in life. Measuring the young trees would lead to an overestimation of site index. Carmean (1956) constructed separate sets of site index curves for Douglas-fir growing on five different soil types. Stage (1963) wrote that differences in vegetative type, genotype, stocking or soil may produce a variety of growth patterns on land of the same site quality. He developed site curves for grand fir that varied with the number of growth rings in the first 1.5 inches radius at breast height.

There has been continued interest in the use of volume rather than height in site classification. Mader (1963) wrote that volume and height growth may not be highly correlated. He quotes Cajander, "an increase of height and an increase in volume, as is proved, do not develop along parallel lines; on the contrary, from a certain lower site quality upwards the volume of wood may increase while the height remains unchanged."

Diameter growth of a tree or basal area growth of a stand as well as height growth may vary by site. A better site can support a greater basal area. Since stand volume is a function of both basal area and height, height alone may not be a sensitive enough measure of volume production.

Perhaps the most frequently voiced objection to the use of height in classifying sites is that height growth does not seem to be independent of stand density after all. Gaiser and Merz (1951) reported that white oak showed increased height growth in denser stands. They developed site index curves for four stand densities. Lynch (1958) found
that dense stocking led to reduced height growth of ponderosa pine, especially on poor sites. This resulted in underestimation of site quality. Reduced height growth in overstocked stands has also been documented for lodgepole pine (Alexander et al., 1967), slash pine (Collins, 1967), Douglas-fir (Curtis and Reukema, 1970) and western larch (Schmidt et al., 1976).

Local volume tables are based on the assumption of a site-specific height-diameter relationship. As early as 1917 Watson developed a set of site curves for sugar pine that was based on height and diameter. He assumed a fixed relationship between diameter and age, so that age and diameter could share an axis on his graph. In 1932 Trorey proposed height-diameter site curves based on a quadratic equation and the assumption that the maximum height attained by a tree is a function of site quality. In 1940 Meyer suggested that a height-diameter site index might be appropriate for uneven-aged stands. McLintock and Bickford (1957) developed site index curves for uneven-aged stands of red spruce following Meyer's proposal. An index diameter, 14", was chosen, and the site index was height at 14" dbh. The taller a tree was at a given diameter, the better the site. This index had an advantage over conventional site index in that it was not biased by the suppressed juvenile growth of this shade tolerant species, and thus could be used for uneven aged stands. It was created in the same way as early anamorphic height-age site curves however. Heights and diameters were sampled on a number of plots, one curve was fitted to the data, and then a series of curves was developed from the guide curve. The model was not extensively tested
to see if the curves in fact reflected potential volume productivity of a site, or if the shape of the curves conformed to actual growth patterns.

Stout and Shumway (1982) further developed this concept. They developed a series of height-diameter curves for several eastern species. Conventional site index was used to differentiate between site classes, and the curve for each site class was developed independently. They found that, for the species they studied, the height-diameter relationship could be used successfully to predict site index.

This study tests the hypothesis that height-diameter curves vary with site quality for a western conifer, *Larix occidentalis*.
POSSIBLE EQUATIONS TO DESCRIBE
THE HEIGHT-DIAMETER RELATIONSHIP

Height-diameter equations are reviewed by Curtis (1967). Early
height-diameter curves were drawn freehand through plotted data (Ker and
Smith, 1955) but this method was time consuming and produced results
that could not be duplicated. In 1932 Trorey proposed using a quadratic
equation to produce height-diameter curves. He suggested that the inter­
cept be set at 4.5 since diameter is measured at 4.5 feet above the ground,
and that only the ascending portion of the curve should be used. If dia­
meter growth continued beyond this point, height would be assumed constant.
The equation can be solved to find the height and diameter at the highest
point on the curve:

\[ H = 4.5 + bD - cd^2 \]

\[ D_{\text{max}} = b/2c \]

\[ H_{\text{max}} = (b^2/4c) + 4.5 \]

Trorey found that the relationship between the coefficients b and c was
species specific. Once determined, a set of curves could be constructed
to correspond to a series of maximum heights, and these curves could be
related to traditional site index curves.

Ker and Smith (1955) did more work with the quadratic expression of
height-diameter curves, and found that for accuracy and ease of applica­
tion it was better than a number of other height-diameter equations.
Curtis (1967) used least squares regression to fit a number of height-diameter equations to Douglas-fir data and found that a quadratic equation produced curves that did not approach the origin if the intercept was left unfixed, while if the intercept was set at 4.5 feet the standard error increased and maximum height was reached at too small a diameter. He suggests that a curve form that approaches a maximum asymptotically might be more appropriate.

A semi-logarithmic equation, \( H = a + b \log D \), was proposed by Hendriksen in 1950 (Ker and Smith, 1955). Myers (1969) reported that he found this equation to be more useful than others for ponderosa and lodgepole pine.

In 1939 Schumacher, working with volume and age, proposed an equation for forest growth studies based on the assumption that percent volume growth declines with age. This could be expressed as a differential equation: \( \frac{dV}{V} = k \frac{d}{(1/\text{Age})} \). Integration and manipulation of this equation led to the form \( \log V = a + b(1/\text{Age}) \), where "a" is the logarithm of the maximum volume attained. This equation is often used to produce site index curves, with \( \log H \) as the dependent variable and the reciprocal of age as the independent variable (Husch and others, 1972). In the form \( \log H = a + b(1/D) \), it has been used to produce height-diameter curves (Curtis, 1967). The underlying assumption seems to apply to height and diameter as well as to volume and age.

In 1881 Greenhill published a paper discussing tree height and diameter from a theoretical standpoint. He considered the modulus of elasticity of the wood and concluded that in order to maintain stability
the base of a tree "must increase as the 3/2 power of the height, which accounts for the slender proportions of young trees, compared with the stunted appearance of very large trees." In 1917 D'Arcy Thompson published his book *On Growth and Form*, which introduced the study of proportional growth rates. Most of his many examples are zoological, but he did include a discussion of Greenhill's work. He related Greenhill's results to Galileo's principle of similitude, pointing out that the laws which limit tree height are the same as those which account for the bending over of a cat's erect tail. A cat's tail grows beyond the limits of vertical stability; a tree will not.

Several years later Huxley (1924) suggested that the formula $Y = bX^k$ could be used in general to study differential growth. This became known as the allometric formula. The allometric formula does not contain time. It expresses the size of one part of the organism in terms of the size of another part or the whole. Pearsall (1927) showed that if the growth rates in time of two plant parts are logarithmic, then the size of one will be proportional to the size of the other raised to a power that is the ratio of the two logarithmic growth rates. For example, if $\log X = a \log \text{Age}$, and $\log Y = b \log \text{Age}$, then $\log \text{Age} = (\log Y)/b$. Substituting, $\log X = (a/b) \log Y$, or $X = Y^{a/b}$. Doerner (1965) and McMahon (1975) used this equation to study the height-diameter relationship of trees.

When Meyer (1940) proposed his height-diameter site index, he suggested using the monomolecular growth function: $H = 4.5 + S \left(1 - e^{-bD}\right)$. This equation was developed from the assumption that the growth rate of an organism is proportional to the amount of growth yet to be achieved.
(Draper & Smith, 1981.) If the maximum size of an organism is $S$, and $X$ is its size at time $t$, then $\frac{dX}{dt} = b \ (S-X)$. Integrating this equation produces the monomolecular equation, $X = S(1-e^{-bt})$. The curve approaches a maximum, $S$, asymptotically. The rate coefficient, $b$, controls the steepness of the curve. The curve has no point of inflection. Meyer's height-diameter equation is a slight modification of this. Substitution of diameter for time seems reasonable if height and diameter are allometrically related. Insertion of $4.5$ forces the curve through $(0, 4.5)$.

Von Bertalanffy developed a growth model of the form $W = (A^{1-m}-be^{-kt})^{1/(1-m)}$ (Richards, 1959). This model was developed specifically to describe weight or volume growth of animals, and is based on the allometric formula and assumptions about food uptake and use. He hypothesized that the parameter $m$ should fall between $2/3$ and $1$. Richards (1959) suggested that the equation was more widely applicable, and that the restrictions on $m$ should be removed. He used the equation to describe plant growth, and also suggested that it could be used to describe the growth of any part of an organism that is allometrically related to the weight or volume growth of the whole organism. The equation produces a very versatile curve. As $m$ approaches zero, the equation reduces to the monomolecular equation. For values of $m$ larger than zero, the curve is sigmoid. Maximum growth rate, or the point of inflection, occurs at $A \ m^{1/(1-m)}$, a proportionally larger value for larger values of $m$. The relative growth rate at the point of inflection is $b/m$.

Moser and Hall (1969) and Pienaar and Turnbull (1973) used this model in forest growth studies. Recently variations of this equation
have become popular for use in polymorphic site curve construction. It is often referred to in forestry literature as the Richard's function.

In this study, each of these equations was fitted to western larch tree height and diameter data by site class in order to select a useful equation for preparing height-diameter site curves.
METHODS

Height-diameter curves were developed for each of five site classes, and the curves compared to learn if the height-diameter relationship varied with site quality. Site classes were comprised of 10 foot site index intervals, using conventional height-age site index, base age 50. Site class 1 included trees with a site index of 70 or above, site class 2 included trees with a site index between 60 and 70, and so on. Site class 5 contained all trees with a site index less than 40.

Several height-diameter equations were fitted to the data by site class using least squares regression in order to select an equation which provided an adequate fit over the range of the data. Examination of residuals and comparison of the magnitude of standard errors of regression provided criteria for selection of a model. Linear equations tested include a quadratic equation; a semi-log equation, \( H = a + b \log D \); the Schumacher expression, \( \log H = a + b \frac{1}{D} \); and the logarithmic form of the allometric equation, \( \log H = a + b \log D \). Dummy variables were used to test for significant differences between site classes. Nonlinear models used were the monomolecular equation and the Richard's equation. The monomolecular form, \( H = 4.5 + S(e^{-bD}) \) was fitted to the data with \( b \) fixed and not fixed to test Meyer's hypothesis that the parameter is a function of species and does not vary with site. The Richard's function was also applied with and without parameter constraints to learn whether the shape of the growth curve varied with site quality.
Finally, a family of height-diameter curves was developed from a single equation, a variant of the Richard's function, following the methodology used for polymorphic site curve construction by Ek (1971), Payandeh (1974), Carmean and Hahn (1981) and others. Results produced by this method were compared with polymorphic curves produced independently from each other.

Data used came from three sources. The U.S.F.S. Intermountain Experiment Station provided access to data collected by Cummings in the 1930's. Some of the data came from the U.S.F.S. Forest Product Laboratory's Western Wood Density Survey. Additional data were collected in the field. Field work for the latter was done in the fall of 1981 on the Lolo National Forest. Height and diameter measurements of western larch trees growing in pure or mixed stands were obtained, and site index estimated for each stand. Height was measured to the nearest foot using a clinometer. Diameter was measured at breast height to the nearest .1 inch with a diameter tape. Site index was estimated by inserting measured height and age into Brickell's (1970) formula:

\[ SI = 0.37956 H e^{(48.4372 \text{ Age})} \]

An effort was made to select young stands as data on small trees was least prevalent in the existing data.

The three sources combined included data for 1369 trees. Ages were not available for every tree, but ranged from 10 to over 300 years. Heights ranged from 5 to 200 feet, and diameters from 0.5 to 45 inches. Site index ranged from 28 to 80 feet at 50 years.
RESULTS

LINEAR MODELS

All linear equations showed significant differences between sites ($\alpha = .05$), with trees on better sites reaching greater heights at a given diameter than trees on poor sites. This relationship did not hold for small trees however. Not all equations provided a completely adequate fit.

The Quadratic Equation:

Trorey suggested that for the height-diameter equation

$$H = 4.5 + bD - cD^2,$$

$b$ and $c$ are linearly related for a given species, and that once this relationship is quantified for a species, no further data is necessary to construct a height-diameter curve to correspond with any maximum height. This would make the equation particularly useful for construction of height-diameter site curves, as maximum height is strongly correlated with site quality. For this set of curves, the relationship was $b = 8.42 - 3.57c$. The relationship between $b$ and $c$ was not a strong one however ($r = - .46$), indicating that Trorey's hypothesis may not be correct for this species.

Examination of residuals showed that the quadratic equation fit the data well. The observed values seemed to be randomly distributed around the regression lines.

Solving for maximum height and diameter at which maximum height was reached produced results consistent with observed values. (See Table 1.)
Table 1. Estimated maximum heights and diameters at which maximum heights are reached.

<table>
<thead>
<tr>
<th>Site Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{\text{max}}$ (feet)</td>
<td>178</td>
<td>165</td>
<td>147</td>
<td>130</td>
<td>111</td>
</tr>
<tr>
<td>$D_{h_{\text{max}}}$ (inches)</td>
<td>42</td>
<td>40</td>
<td>35</td>
<td>31</td>
<td>28</td>
</tr>
</tbody>
</table>

The Semi-log Equation:

The semi-log equation, $H = a + b \log D$, produced inadequate results. Examination of the residuals showed a marked z-shaped pattern. In addition, the curves do not approach zero, but predict negative heights for very small diameters. The curves do not approach an upper limit, so cannot be used to predict maximum height.

The Schumacher Equation:

This equation, $\log H = a + b(1/D)$, also produced inadequate results. Based on a very simple growth model, it produces an s-shaped curve. Plots of $\log H$ against $1/D$ showed a strong deviation from the main trend for very small trees. Because of this, only trees with diameters greater than five inches were considered.

This model predicted maximum heights greater than observed maximum heights for all sites. Examination of residuals indicated that the model did not provide a completely adequate fit of the data.

The Allometric Equation:

This equation $H = aD^b$ or $\log H = \log a + b \log D$ resulted in an extremely poorly fitting set of curves. Plots of the transformed data indicate that $\log H$ and $\log D$ are not linearly related through the range
of the data, or to use Huxley's (1924) term, there is not a constant
differential growth ratio. Instead, there is at least one change in
trend, and possibly a gradual continuous change in trend.

Examination of the plotted data suggested that the shift took place
between 10 and 20 inches in diameter. The plots also suggested that the
shift took place earlier on poor sites than on good sites; that is, on
poor sites trees showed reduced height growth at a smaller height than
on good sites. Possibly trees follow the same initial growth pattern on
all sites. Differences between sites among larger trees might be due to
their relative sizes when the initial trend is substituted with the trend
of reduced height growth.

To explore this possibility further, trees were segregated by size.
Considering only trees with diameters less than 20 inches, regression of
log H on log D with dummy variables for site showed no significant
difference between sites (α = .05) with the exception of site 3. Predic-
ted values for site 3 and the other sites combined were never more than
three feet apart in the range of diameters from 5 to 15 inches. Selection
of diameters greater than 20" resulted in five separate regression
lines, one for each site class. These lines were plotted together with
the line for small trees, and the intersection points noted (see Figure 1).
Figure 1. "Segmented" regression lines of log H on log D by site.
Examination of residuals showed that these regression lines provided a much better fit than that obtained by fitting one line to each site class across the range of the data, although the data shift more gradually than the abrupt intersection of the lines indicates.

**NONLINEAR MODELS**

Parameters for the nonlinear models were estimated using the BMDP derivative-free nonlinear regression program BMDPAR. Estimates were obtained separately for each site class.

The Monomolecular Equation:

The monomolecular equation, \( H = 4.5 + S(1-e^{-bD}) \), produced curves which conformed fairly well to the data. Predicted maximums were approximately equal to observed maximums for each site. For the better sites (site classes 1 and 2) the residuals showed that the curves overestimated heights of small trees and underestimated heights in the mid-range, indicating that an s-shaped curve might be more appropriate than the monomolecular.

When Meyer (1940) proposed using the monomolecular equation to produce height-diameter site curves, he hypothesized that the parameter \( b \) was species dependent and would not vary with site, while \( S \), maximum height, was a function of site. McLintock and Bickford (1957) in their work with red spruce, assumed this to be true and estimated \( b \) for their entire data set. Stout and Shumway (1982) working with several deciduous species, grouped their data by site and found that \( b \) varied with species but for a given species did not vary with site, confirming Meyer's hypothesis. If \( b \) is constant for a species, the equation is especially use-
ful as it can be solved for $S$, $S = (H - 4.5)/(1 - e^{-bD})$. Maximum height and site quality are strongly correlated; measured heights and diameters can be substituted into the equation to produce a direct estimate of maximum height and site quality.

Results of this study suggest that, for western larch, $b$ is not constant, and polymorphic curves are needed. Examination of the magnitude of change in $b$ from one site to the next and the standard deviations of the estimates of $b$ (shown in Table 2) indicate that the differences in $b$ are not insignificant. To confirm this, $b$ was estimated for all sites combined, producing a value (.0523) between that of site 2 and site 3. Then the regression was run again by site with $b$ fixed at this value. This produced a set of anamorphic curves (see Figure 2).

<table>
<thead>
<tr>
<th>Site Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>200</td>
<td>180</td>
<td>154</td>
<td>131</td>
<td>113</td>
</tr>
<tr>
<td>st dev $S$</td>
<td>8.05</td>
<td>6.50</td>
<td>4.14</td>
<td>3.79</td>
<td>2.84</td>
</tr>
<tr>
<td>$b$</td>
<td>.0466</td>
<td>.0506</td>
<td>.0658</td>
<td>.0835</td>
<td>.0871</td>
</tr>
<tr>
<td>st dev $b$</td>
<td>.0035</td>
<td>.0037</td>
<td>.0045</td>
<td>.0062</td>
<td>.0047</td>
</tr>
</tbody>
</table>
Figure 2. Height-diameter curves by site obtained by fitting the two forms of the monomolecular equation.
The anamorphic curves produced an overestimation of height for large
trees on poor and medium sites, and a slight underestimation on the best
sites. Observed maximum heights correspond more closely to those
estimated by the monomolecular equation when b is not fixed. Examination
of residuals around the anamorphic curves showed a strong non random
distribution especially on poor sites. The standard errors of the
regression lines were also slightly increased when b was held constant
(see Table 3).

Table 3. Two forms of the monomolecular equation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Site Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>--------------</td>
<td>---</td>
</tr>
<tr>
<td>Standard Error of Regression</td>
<td></td>
</tr>
<tr>
<td>b unfixed</td>
<td>11.6</td>
</tr>
<tr>
<td>b fixed</td>
<td>11.7</td>
</tr>
<tr>
<td>Estimate of Maximum Height</td>
<td></td>
</tr>
<tr>
<td>b unfixed</td>
<td>204</td>
</tr>
<tr>
<td>b fixed</td>
<td>194</td>
</tr>
</tbody>
</table>

The change in b with site indicates that curves for poor sites level
off sooner than those for good sites. A tree of a given diameter on a
poor site has reached a greater proportion of its total height than a
tree of the same diameter on a good site, even though it is shorter. For
example, a 20" tree on a site 1 will be 126' tall, 60% of its maximum
height, while a 20" tree on a site 5 will be 97' tall, 82% of its maxi-

mum height.
Professor Loftsgaarden has pointed out that if the 4.5 intercept is accounted for in the difference equation before integrating to produce the monomolecular equation, the following equation results: \( H = 4.5e^{-bd} + S(1-e^{-bd}) \). This equation has a 4.5 intercept and approaches a maximum value of \( S \) instead of \( S + 4.5 \). This equation produced curves virtually identical to those produced using the simpler form.

The Richard's Equation:

To explore further the polymorphic nature of the curves, Richard's (1959) modification of Von Bertalanffy's growth model was applied to the data. This four parameter model takes the form \( H = S(1-ke^{-bd})^{1/(1-m)} \). The parameter \( k \) is determined by initial tree size; to produce curves that go through the origin \( k \) was set to one. \( H - 4.5 \) was used as the dependent variable to compensate for the zero intercept.

The three parameters \( S \), maximum height, \( b \), and \( m \) were estimated for each site. The resulting curves fit the data well. Sites 1 and 2, which were not adequately described by the monomolecular equation, showed no evidence of lack of fit, and the standard errors of these two regression lines were smaller than those of the monomolecular equation.

The values obtained for \( S \) were similar to those of the monomolecular equation. No pattern of increase of decrease across the sites could be discerned for parameters \( b \) and \( m \) however. (See Table 4.) This is a disadvantage of increasing the number of parameters in a nonlinear model.
Nearly identical curves can be produced with quite different parameter values. This makes it hard to compare and interpret parameter estimates from one case to another. The problem of overparameterization is discussed by Zeide (1978).

Table 4. Parameter estimates and standard deviations for the three parameter Richard's model.

<table>
<thead>
<tr>
<th>Site Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>175</td>
<td>159</td>
<td>150</td>
<td>124</td>
<td>107</td>
</tr>
<tr>
<td>St dev S</td>
<td>6.1</td>
<td>3.9</td>
<td>4.7</td>
<td>3.3</td>
<td>2.7</td>
</tr>
<tr>
<td>b</td>
<td>.083</td>
<td>.097</td>
<td>.081</td>
<td>.126</td>
<td>.119</td>
</tr>
<tr>
<td>St dev b</td>
<td>.012</td>
<td>.011</td>
<td>.013</td>
<td>.02</td>
<td>.015</td>
</tr>
<tr>
<td>m</td>
<td>.37</td>
<td>.454</td>
<td>.174</td>
<td>.34</td>
<td>.233</td>
</tr>
<tr>
<td>St dev m</td>
<td>.094</td>
<td>.07</td>
<td>.123</td>
<td>.118</td>
<td>.092</td>
</tr>
</tbody>
</table>

Because the inclusion of $m$ was necessary to produce an s-shaped curve, $b$ was estimated for the entire data set and fixed. The resulting 2-parameter model, $H=4.5=S(1-e^{-0.076D})^{1/(1-m)}$, produced curves very similar to the 3-parameter model. The residuals gave no indication of lack of fit and the standard errors were virtually the same as those of the 3-parameter form. (See Table 5.)

Table 5. Standard errors of regression of the two parameters and three parameter Richard's function.

<table>
<thead>
<tr>
<th>Site Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-parameter</td>
<td>10.9</td>
<td>9.1</td>
<td>9.5</td>
<td>10.4</td>
<td>6.2</td>
</tr>
<tr>
<td>2-parameter</td>
<td>10.9</td>
<td>9.7</td>
<td>9.1</td>
<td>10.8</td>
<td>6.6</td>
</tr>
</tbody>
</table>
The parameter \( m \) controls the steepness of the curve. Large values of \( m \) produce a steeper curve, as do large values of \( b \) in the monomolecular equation. Values of \( m \) greater than zero produce an s-shaped curve.

Table 6. Parameter estimates of the two parameter Richard's model.

<table>
<thead>
<tr>
<th>Site Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>178</td>
<td>167</td>
<td>151</td>
<td>135</td>
<td>117</td>
</tr>
<tr>
<td>( m )</td>
<td>.313</td>
<td>.304</td>
<td>.127</td>
<td>-.034</td>
<td>-.062</td>
</tr>
</tbody>
</table>

The magnitudes of both parameters were positively correlated to site quality (see Table 6). Sites 1, 2 and 3 show maximum growth rates at heights of 32, 30 and 14 feet, respectively, while sites 4 and 5 show constantly declining rates of height growth with respect to diameter. Curves for poor sites level off at a smaller diameter than those for good sites, as did the curves produced with the monomolecular equation. (See Figure 3.)

**CREATING A FAMILY OF CURVES**

If the series of height predictor equations can be modified into one equation with site as an independent variable, and if this equation can be solved for site, there would be no need to interpolate from a graph to estimate site index directly from field measurements. If the site curves are anamorphic this is easily done. A popular procedure is outlined by Husch and others (1972). If the curves are polymorphic this is not so easily done. Polymorphic curves present special problems in interpolation.
Figure 3. Height-diameter curves produced using the Richard's function with two parameters.
In order to develop a single equation that would produce a family of polymorphic curves, a number of investigators have used variations of the Richard's equation. Typically, the form of the equation used to produce polymorphic site curves is \( H = b_1 S_l b_2 (1-e^{-b_3 Age}) b_4 S_l b_5 \). Because the equation cannot be solved algebraically for site index, several researchers (Payendeh, 1974; Monserud, 1978; Carmean and Hahn, 1981) have fit another regression line to the same data: \( S_l = b_1 H b_2 (1-e^{-b_3 Age}) b_4 H b_5 \). The two expressions are not equivalent however. As Curtis and others (1974) point out, even if the same data set is used, the substitution of one dependent variable for another involves an entirely new sum of squares. This makes the benefit of developing a family of curves instead of a set of separately estimated curves questionable.

The five parameter model was fitted to the data, as well as several equations with one or more of the parameters set to one. A four parameter equation \( H = b_1 S_l b_2 (1-e^{-b_3 Age}) b_4 S_l \) produced curves nearly identical to the five parameter equation and resulted in an increase of only 0.1 feet in the standard error. Fewer parameters were insufficient to produce the needed polymorphism. The four parameter model was also developed with site index as the dependent variable. The resulting equation showed a strong tendency to overestimate site index for small trees and underestimate it for large trees. If the site index must be calculated numerically instead of estimated from a graph or table, perhaps it should be done by the procedure outlined by Ek (1971) who suggested that an iterative process could be performed in a computer subroutine to try several site index values in the height predictor equation until a value
close to the true height was obtained.

**SUMMARY**

The semi-log equation, the Schumacher equation and the allometric equation were clearly inadequate for describing the height-diameter relationship. The quadratic equation, the monomolecular function and the Richard's expression all produced curves that fit the data well. Values of $r^2$ for all sites were greater than .95. (For comparative purposes a statistic comparable to an $r^2$ was computed for nonlinear estimates.) The quadratic equation is the least interesting of the three. Although all equations are used empirically, it seemed desirable to select an equation based on a growth model. The parameter values of both the monomolecular and the Richard's function can provide insight into tree growth patterns.

The two nonlinear models produced curves which were virtually indistinguishable at diameters greater than 20". Below this point the monomolecular equation was less adequate than the Richard's because it could not produce an s-shaped curve. The Richard's equation provided the best fit across the range of diameters and sites.

The curves produced by the Richard's function cross in the lower diameter range. Separate examination of the smaller trees using the allometric formula indicated that the initial height-diameter growth pattern was the same on all sites. These curves are not useful as site index predictors for stands where diameter is less than 20". This is a major limitation of their usefulness, and restricts their application to old growth stands. This does not imply however that small trees grow as fast
on poor sites as they do on good sites, just that they grow the same shape.

The growth and yield tables prepared by Schmidt and others (1976) substantiate these results, although they indicate a separation of height-diameter curves at a slightly smaller size than these results. Curves produced by extracting values from the height-age table and the diameter-age table indicate a height-diameter growth trend essentially the same for all sites, with some crossing of the curves at small diameters. The time rate of growth does vary by site however. For example, the tables show that a dominant tree with a site index of 80 will be 63 feet tall when it is 8.9 inches in diameter. A dominant tree with a site index of 50 will be 59 feet tall when it is 9.0 inches in diameter. The heights and diameters are very close, but the tree on the better site is 20 years younger than that on the poor site.

The standard errors of the regression equations are rather large, but each curve represents a mean for a range of sites. It is appropriate for some of the observations to be scattered away from the curves.

The set of curves produced by the Richard's function can be approximated from the equation: \( H + 4.5 = 13.4S.1.59(1 - e^{-0.1015D})0.28S.1. \). The standard error of this estimate is 9.7 feet. In the upper diameter range this set of curves could be used to estimate site quality from measured tree heights and diameters.
DISCUSSION

Results indicate that the initial height-diameter growth pattern of relatively rapid height increase is followed by trees on all sites. At some point height growth is reduced, while diameter growth continues. That this point occurs at a smaller size on poor sites is indicated by the shape of the polymorphic height-diameter curves.

Effects of stocking levels may account for the large variability of heights within site classes for small trees. The effects of stocking may mask the effect of site. By the time trees reach the relatively large size at which height-diameter curves differentiate, the stand may have overcome initial stocking effects, and approached a normal stocking level. This "trend toward normality" is discussed by Davis (1966).

Stout (1967) has suggested that the substitution of the initial trend of rapid height growth with respect to diameter by a trend of reduced height growth may occur because crowns of small trees can capture light at a rate proportional to their volume, while crowns of large trees lack leaves in the center and capture light at a rate proportional to their surface area. The surface area of the crown can increase most rapidly by increasing width. Loss of interior foliage presumably takes place when the tree is a given size regardless of site quality. If a tree's shape is, as suggested, the result of the efficient allocation of biomass for the purpose of increasing photosynthetic capability, the shift to a trend of reduced height growth on better sites may be delayed
by the closed condition of the canopy. A good site can support a greater basal area than a poor site. By the time trees reach the size at which the height-diameter curves begin to differentiate, the canopy on a poor site may be somewhat open, while remaining fully closed on a good site. Trees growing in the more open condition could increase photosynthetic area by increasing crown width, causing a relative increase in diameter growth, while trees in a closed stand might better increase photosynthetic surface by growing upward.

Another possible explanation of the early reduction in height growth on poor sites has to do with priorities of carbon allocation. Photosynthate must first be used for respiration and maintenance; excess is available for growth. Possibly, in cases where surplus photosynthate is limited, diameter growth takes precedence over height growth. Schmidt (1980) found that height growth of western larch occurs later in the growing season than diameter growth. Trees on poor sites may have little surplus photosynthate available for height growth, especially as they get larger and respirational demands increase.
CONCLUSION

If an independent, accurate measure of site quality were available, the height-diameter-site relationship might be more fully explored. Since the height-age relationship, site index, was used to estimate site quality, the possible interactions of height, diameter and age cannot be fully explored. Results did indicate that the height-diameter relationship could be used to predict site index for large trees. For trees smaller than 20 inches in diameter, the height-diameter relationship was not a successful predictor of site index. This may be because site has no influence on the height-diameter relationship of small trees, or because the influence of site is hidden by as yet unexplained variation.
LITERATURE CITED


37


Greenhill, A. G. 1881. Determination of the greatest height consistent with stability that a vertical pole or mast can be made, and of the greatest height to which a tree of given proportions can grow. Proc. Cambridge Phil. Soc. IV Part II: 65-73.

Husch, B. 1956. Use of age at dbh as a variable in the site index concept. J. For. 54: 340.


