Operational management techniques for inventory control

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OPERATIONAL MANAGEMENT TECHNIQUES
FOR INVENTORY CONTROL

By

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Chairman, Board of Examiners
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TABLE OF CONTENTS

CHAPTER PAGE
I. INTRODUCTION ............................................ 1
II. DEMAND FORECASTING ...................................... 6
III. ORDER QUANTITY ............................................ 20
IV. ORDER SCHEDULING ........................................ 31
V. WESTMONT TRACTOR COMPANY ................................ 35
VI. CONCLUSION ................................................ 40
BIBLIOGRAPHY .................................................. 44
CHAPTER I

INTRODUCTION

Perhaps everyone at one time or another has innocently entered a store expecting to buy some rather common item only to be told that the item in question is out of stock. Sometimes the sales clerk will even go so far as to suggest that the customer wait while the item is ordered or a new shipment arrives. If the customer apologetically mentions that he has an immediate use for the item, the clerk may condescend to direct him to another store where the item is in stock. Actually, the customer need not feel apologetic about his demands, for the American economy produces a wide number of products on a "made to sell" basis. Competition at the retail level dictates that the typical retail establishment must be ready to satisfy the consumer's immediate demands. Consumers have come to expect retail establishments to offer an ever-growing variety of products as the economy continues to expand and develop. They are no longer content to accept many products on a "made to order" basis as they may have been a couple of decades ago.

The increasing complexity of the American economy with its tremendous variety of products and the more demanding expectations of the buying public are important reasons for the increased interest and study in the field of inventory control. As economic forces have come to dictate the necessity of large investments in inventory, it has become apparent to many businessmen that there is a need for a body of knowledge and techniques to help cope effectively with the problems
involved in the handling and control of inventories. Although inventories perform an important function at many stages of the manufacturing and distribution process, this paper will be primarily concerned with the subject of inventory control at the retail level.

The writer is convinced that ineffective inventory control at the retail level is a potential cause of business failure. The small retail operator may underestimate the size of investment required in inventory, thereby losing customers to other firms offering a wider variety of products. His record keeping system may be defective, resulting in surpluses and shortages, or he may have excessive capital tied up in inventory that could be better used in other parts of the business. This paper presents some concepts and analytical procedures which the writer believes to be of practical value for managers of many typical retail firms.

As with many other areas of knowledge it seems to be impossible to completely isolate the subject of inventory control from other aspects of business such as marketing, finance or management. In fact, probably most of the various aspects of business have some bearing upon decisions affecting inventories. For example, demand forecasting freely utilizes marketing principles and information. The amount of investment in inventory may be determined in accordance with the financial policies and needs of the firm as a whole. Accounting techniques are used to ferret out some of the hidden costs involved in carrying inventory. The point is simply that the writer makes no attempt to define the limits of the subject of inventory control as a formal field of study; such discussions will be left to the more philosophically inclined.
It might be convenient to think of inventory control procedures as mainly concerned with performing three general functions: demand forecasting, deriving order quantities and order scheduling. The purpose of these functions, of course, is to minimize investment and carrying costs in relation to any given level of customer service. Of the three functions mentioned, demand forecasting is usually the first step in the decision process. The demand forecast is converted into a usage rate on a daily, weekly or monthly basis, or any unit of time that is convenient. The usage rate is a necessary input into almost any formula for deriving order quantities or order scheduling.

The formulas and techniques discussed in this paper are primarily concerned with what is often called the "dynamic inventory problem under risk." That is, demand for a product continues indefinitely with a certain amount of random variation. The writer believes that most firms are built around such items. Of course, there are many products which are "one shot" items. Their life cycle is limited to a few weeks or months, and if the stock of such items is not sold during that time, they are reduced to salvage value. Examples of such items would be seasonal fashions in clothing, Christmas trees, perishable foods, etc. Beyond a certain period of time, excess stock must be disposed of at an amount below their original cost. For the ambitious manager who is mathematically inclined there are methods of constructing "payoff matrices" which yield order sizes for giving best payoff over the long run.1

Probabilities are assigned either subjectively or from historical

---

records, to a range of possible demands which might be realized during the economic life of the product.

These probabilities are then multiplied by gains or losses which are produced by the various combinations of order quantity and corresponding demand levels. The losses or gains are found by comparing the value of various order size decisions with the value of a perfect prediction. That is, the total return when order size is equal to realized demand. For example, if the loss from choosing a certain order quantity is 10 dollars when a certain other level of demand is actually realized and that level of demand has a 10 percent probability of happening, then the expected opportunity loss over the long run would be 10 percent of 10 dollars or one dollar. The expected opportunity losses for a range of possible order quantities corresponding with a certain demand level is calculated. Similarly, a range of opportunity losses corresponding to each of the other possible demand levels is also calculated. The most profitable order size is, of course, the one which has the lowest expected opportunity loss. In order to be valid, however, this decision rule must be obeyed over the long run of such decisions. The writer feels that the above described system is best suited for those firms which can afford the services of a large high-speed digital computer. Due to the complexity of the system, it is probably of little value to small independent type retail firms; therefore, it will not be discussed any further in this paper.

The classical economic order quantity theory discussed in chapter three is based on the assumption that demand is known with certainty. Of course, demand is seldom known with certainty; however, the writer
believes that many forecasting techniques approximate demand close enough to allow the assumption of certainty to be relaxed in applying the theory to many practical business situations.
CHAPTER II

DEMAND FORECASTING

It is probably impossible to make any kind of meaningful inventory decision without some sort of estimate of future demand, regardless of how difficult it is to make such an estimate. In fact, investing in some product for which there is no tangible basis for estimating its demand over a period of time is usually for the purpose of speculation. The inventory manager may be able to get by without finding economic order quantities or the most efficient order scheduling system, but the decision to invest in a certain product usually carries at least an implied estimate of demand for that product.

Demand forecasts tend to fall into two general categories: forecasts based on past observations of demand for the item itself, and forecasts based upon external events which in some way influence the demand for the item.\(^2\) In the first category, total demand for the item is systematically recorded on a weekly, monthly or some other periodic basis so that the forecast is simply a projection of its own past history. Various techniques may be used to detect trends or gradual changes in the level of demand so that such trend may be projected into the future. In the second category of forecasts, the demand for an item is linked to some external event. The field of marketing has

established many such relationships. For example, the demand for many luxury items can be correlated to national statistics for disposable income. Other national indexes are often correlated with various classes of products. If the index is predicted to increase by a certain amount, the demand forecast is increased accordingly. Other external factors may include competition, taste and styles, promotional efforts and the distribution channels. In many cases experience with the product provides the inventory manager with an understanding of its relationship to external factors. For example, the Westmont Tractor Company (details of Westmont Tractor's inventory system described in Chapter V) recently hired a parts salesman to fill the position of inventory manager. The general manager explained that the inventory manager's experience in parts sales had been one of his major qualifications for the job. Through his experience in parts sales the inventory manager learned which parts and approximately how many a particular piece of equipment would need during its useful life. By keeping track of approximately how many pieces of equipment are currently in use in the trade area, he is able to make an estimate of total demand for many of the parts that will be needed. His experience becomes particularly important in the event of a significant change in demand, such as might be caused by the initiation of a major highway project which would bring a number of new pieces of equipment into the area. Although demand forecasts for many of the parts stocked by Westmont Tractor are derived from historical data, a development such as a new highway project is reason for the inventory manager to review the demand estimates for those parts which he feels will be affected by the project.
The type of forecast which is primarily based upon historical data is useful for any product that has a continuing demand with relatively small variations over a period of time. Obviously historical data is of little use if the demand for a product varies randomly by two or three hundred percent from one period to the next. This is not to say, however, that some random variation will nullify the usefulness of historical data for statistical measures may be used to assign probabilities to the various levels of demand that have been recorded for a particular item. On the following page is a copy of a stock history card taken from the Inventory Procedures manual used by the Westmont Tractor Company. This stock history card is a typical example of a historical record because it takes into consideration most of the essential facts which must be known about the particular part it represents. For example, it gives the name of the part plus a brief functional description to help further identify the part. In this case it is a cylinder liner used in five and three-quarter inch bore engines. Below the identification data on the card, the history of demand is shown for each year from 1954 to the first quarter of 1959. The total demand for each year is shown at the bottom of the vertical columns with the horizontal columns breaking the yearly demand down into quarterly periods. The importance of breaking yearly demand down into smaller periods such as quarters or months should be emphasized. Yearly demand for an item might be fairly constant or gradually increasing, as in the case of the cylinder liners; however, quarterly or monthly totals may reveal a pronounced cyclical pattern during the year. Another important point that should be considered is the matter of
**Fig. 1. Stock history card. Records past demand for an item for the purpose of making demand forecasts.**
defining total demand. To simply use the total quarterly sales figure would generally be grossly inaccurate. The Westmont Tractor Company derives a more meaningful figure for total demand by recording all calls for an item whether or not a sale has actually been made; such might have been the case for a customer who called for an item which was out of stock but who was not willing to wait for the part to be back ordered or for a new shipment to arrive. Then it was reasoned that since a shop mechanic may draw many more parts than he will actually need for a particular job and will usually return the unneeded ones for a refund, the number of returns and cancellations should be deducted from the total number of calls for a particular part. Intuitively at least, a formula which deducts returns and cancellations from the total number of calls would seem to give a more realistic picture of demand than just using the figure for total sales.

Once a systematic procedure for recording demand has been developed and which gives a meaningful definition of demand, a number of formulas and techniques may be used to convert the data into a forecast. If demand remains constant with only random variations, that is, the variations are not cyclical or there is no noticeable trend taking place, then a simple average over a representative number of periods can be used as a forecast. Although demand realized during any one period will probably never exactly agree with the forecast, random surpluses during some periods will tend to cancel random shortages during other periods. It should be recognized, however, that a buffer stock must be carried to cover the unpredictable high levels of demand which might occur during any period. The theoretical justification for using an
average of demand levels observed during past periods is the idea that random variations in the level of demand (or random variations in any particular quantity) have a tendency to be normally distributed about the average or arithmetic mean. This means that realized demand for a certain period has a greater probability of lying within an interval which is close to the average than it does of lying within an equal sized interval located farther away from the average. The important thing to remember is to make sure that there is no cyclical pattern in the demand for an item.

Probably the most common example of a cyclical pattern is the seasonal variation felt by many products. For example, lawnmowers obviously experience their peak demand sometime during spring or summer with the low during the fall and winter. In this case, depending on how it was designed, a simple average would have little practical meaning. If the average was computed on the basis of monthly totals over a 12-month period, the average would amount to a figure consistently below the monthly demand during the spring and summer months and consistently above demand during fall and winter months. In other words, the cyclical type of demand pattern presents a distribution of demand levels which does not tend to center about the average of mean, but, rather, tends to look like a bi-modal distribution with two peaks. One peak will represent high demand levels of the spring and summer months and the other peak will represent low demand levels of fall and winter months. The average will lie somewhere between the two peaks, therefore the probability of realized demand for any particular month is less for demand levels lying in an interval close about the average than demand
levels which lie in an equal interval farther away from the average. The point is to choose a figure for the demand forecast which has the greatest probability of being close to realized demand. With a cyclical demand pattern the simple average only has value for forecasting the demand over one complete cycle or multiples of a complete cycle. In case of the lawnmowers one would have to compute the average on the basis of yearly totals, then the average would be a meaningful figure for predicting total demand during the period of a year. A simple solution to the problem of making a monthly forecast or for any period less than the complete demand cycle would be to take the same month from each of a number of years and average their demand levels. If we wanted to forecast the demand for January, for example, we could average the demand levels of each January from a number of past years.

The simple average as stated before is applicable to a situation where demand over the long run has tended to remain constant with only relatively small random variations. If demand has established a trend which is gradual but continuing over a longer period of time, usually a year or more, then a simple average will become increasingly inaccurate as time goes on. If demand is rising, then the demand levels of earlier periods will tend to drag the average down. Although larger demand levels of later periods tend to raise the average when it is computed after each succeeding period, the demand levels from earlier periods will keep the average from rising as fast as actual demand is increasing. This condition may go on for a long time before it is noticed because the random variations in the demand level from one period to the next tend to obscure the presence of a definite trend in the
demand pattern. If the average includes the demand levels from a large number of past periods, the figure for each succeeding period causes only an infinitesimal change in the average. In order to help detect an established trend a moving average may be used. A moving average is simply an average computed on the basis of a certain predetermined number of periods. When the average is computed at the end of each succeeding period the oldest period is dropped, therefore the number of periods remains the same each time the average is computed. The moving average is actually a compromise between the smoothing effects of the simple average and the need to spot trends as soon as possible. The simple average is able to minimize the destabilizing effect of random variations upon the demand forecast. The moving average on the other hand sacrifices some of the stability features of the simple average by taking into account a smaller number of observations. Of course, this same characteristic is what allows the moving average to be more responsive to the presence of a trend. In choosing the number of periods to be included in a moving average, it is important to choose a large enough number of periods so that the average is stable, yet small enough for the average to respond to a trend.

To a lesser degree the moving average still suffers from the same defect as the simple average. It tends to lag behind a trend. If demand were gradually increasing, for example, the earlier periods would drag the average below the current level of the trend. Since the earliest period is dropped each time, the average is computed. However, the moving average will tend to follow a trend rather than becoming increasingly inaccurate as the simple average tends to. If
If a product's demand is increasing or declining more rapidly than what one might consider gradual and the increase or decline is more than just a temporary random variation, then a weighted moving average is perhaps more applicable. For example, many products have a slightly "faddish" appeal to the consumer such as certain styles of clothing. A certain article of clothing may be "in" for six months or some indefinite period of time covering a number of forecast periods, then quickly decline to some fraction of its previous demand for an indefinite period of time. In this case there are actually two important characteristics to contend with: relatively small random variations from one forecast period to the next and random variations of much larger magnitude which span an indeterminate number of forecast periods. These larger variations are not to be confused with any cyclical pattern such as might occur on an annual basis. As one would suspect, the simple average forecast would probably never come very close to the actual demand realized in the forecast period and the moving average lags too far behind a trend of rapidly increasing or decreasing demand. The weighted moving average, however, responds quicker to the onset of a trend because the observations from the most recent periods are given more weight than the earlier periods. For example, if the forecast period was a month and a six period moving average is used for the forecast, a useful way to weight it might be to multiply the most recent period

\[ \text{Ibid., p. 310.} \]
by 6, the next most recent by 5, the next by 4, and so on, so that the first is finally given a weight of only 1. The results of each multiplication are added and the total is divided by 21. That is, the weighted average may be calculated as follows:

<table>
<thead>
<tr>
<th>Weighting factor</th>
<th>Demand/period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>x</td>
</tr>
<tr>
<td>5</td>
<td>x</td>
</tr>
<tr>
<td>6</td>
<td>x</td>
</tr>
<tr>
<td>Total</td>
<td>x</td>
</tr>
</tbody>
</table>

so that, \( \frac{\text{Total}}{21} \) = weighted Moving Average

The value of the weighting factor is largely arbitrary but may be chosen to achieve a greater or lesser degree of responsiveness depending upon what is needed. If demand trends are changing rapidly, then the most recent period should be weighted relatively heavy compared to earlier periods.

The above described method where integer values are used for the weighting factor is called arithmetic weighting. Another type of weighting where fractional values are used is called geometric weighting. For example, if a four-period moving average is used, the latest period could be multiplied by \( \frac{8}{15} \), the next by \( \frac{4}{15} \), then \( \frac{2}{15} \) and the earliest period by \( \frac{1}{15} \). The results of the multiplications are totaled to give the average directly.

\(^{1}\)Ibid., p. 314.
The fractional weighting must add up to one, otherwise the sum of the multiplications will be meaningless. In the example just shown, the weight assigned to the most recent period is $8/15$ or slightly over half of the total weight for all four periods. Such heavy weighting of the latest period would cause the forecast to be very responsive to the onset of a trend. However, it should be remembered that this responsiveness is achieved at the cost of instability resulting from period to period random variations. Any fractional values may be chosen for the weighting factors to give a desired combination of responsiveness and stability, as long as the sum of all of the factors is equal to one. Of course, the main purpose of a weighted moving average is to allow the demands from recent periods to influence the forecast more than demand levels of earlier periods.

Unfortunately, the moving average type of forecast has a serious drawback for a small operator. It is an effective enough method of forecasting, but the amount of work involved is prohibitive for any but the larger firms.\(^5\) At the beginning of each forecast period the demands must be totaled from all of the periods used in the average, then the total must be divided by the number of periods. A weighted average requires a similar amount of work. Obviously a typical retail operator

\(^5\)Ibid., p. 316.
simply is not able to grind through all of the arithmetic this would entail. Even a small store may stock several hundred products.

Fortunately, there is a rather clever way of eliminating much of the arithmetic involved and still come up with a forecast that approximates the results of going through all of the arithmetic. Going back to the four-period example of a weighted factor, it should be noticed that the most recent period carried approximately half of the weight in the average while all of the previous three periods carried the other half of the average. That is, $\frac{8}{15}$ is approximately half of $\frac{15}{15}$.

Instead of going through the process of dropping the first period and adding the most recent period to calculate a new average, one could simply take half of the demand realized during the most recent period and add it to half of the forecast for that same period to come up with the new forecast.\(^6\) This process is called exponential smoothing because the weighting factor of any one period is a constant raised to some power. In this case the constant is $\frac{1}{2}$. The weight given to the latest period is $\frac{1}{2}$ raised to the first power or simply $\frac{1}{2}$, and taking $\frac{1}{2}$ of the forecast in effect gives a weight of $\frac{1}{2}$ to the second power to the next period and $\frac{1}{2}$ to the third power to the next, and so on.

If the forecast period were a month and a four-month weighted moving average were used to develop the forecast, the following example shows how an exponential smoothing forecast might be derived.

\(^6\)Ibid., p. 316.
<table>
<thead>
<tr>
<th>Month</th>
<th>Weighting factor</th>
<th>Recorded demand in units</th>
<th>Weighted demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1/15</td>
<td>x 5</td>
<td>5/15</td>
</tr>
<tr>
<td>February</td>
<td>2/15</td>
<td>x 3</td>
<td>6/15</td>
</tr>
<tr>
<td>March</td>
<td>4/15</td>
<td>x 5</td>
<td>20/15</td>
</tr>
<tr>
<td>April</td>
<td>8/15</td>
<td>x 6</td>
<td>79/15</td>
</tr>
</tbody>
</table>

or 5 and 4/15 units

The weighted forecast for the demand in the month of May is 5 and 4/15 units or when converted into decimals it is approximately 5.30. If the actual realized demand for the month of May turns out to be 7 units, then the forecast for the month of June is simply as follows:

1/2 of forecast for May is: 1/2 x 5.30 or 2.65
Added to 1/2 of observed demand for May or 3.50

Therefore, the forecast for June is 6.15 units. To see how the short method compares with the long method the forecast for the same month is computed as follows:

<table>
<thead>
<tr>
<th>Month</th>
<th>Weighting factor</th>
<th>Recorded demand in units</th>
<th>Weighted demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>February</td>
<td>1/15</td>
<td>3</td>
<td>3/15</td>
</tr>
<tr>
<td>March</td>
<td>2/15</td>
<td>5</td>
<td>10/15</td>
</tr>
<tr>
<td>April</td>
<td>4/15</td>
<td>6</td>
<td>24/15</td>
</tr>
<tr>
<td>May</td>
<td>8/15</td>
<td>7</td>
<td>93/15</td>
</tr>
</tbody>
</table>

Which equals 6.2 units

Therefore, the 6.15 units forecast for the month of June derived by the short method is very close to the 6.20 units given by the longer weighted average method. The difference between the two forecasts is less than one percent. The advantage of the short method is obvious; for all practical purposes it is as accurate as the long method and yet involves far less computations. The forecast for July and each succeeding period
is computed in the same manner and will continue to give approximately
the same results as the long method.

One difference between the exponential smoothing forecast and the
weighted moving average forecast is that exponential smoothing con­tin­ues to be influenced by the beginning periods. In the above example,
the June forecast given by the short method is still influenced by
January, while the weighted moving average excludes the observed demand
of January. However, the actual weight given to January by the short
method is only about 1/2 of 1/15 or 1/30. This is because the weight
given to January in the May forecast was 1/15 and only half of the May
forecast was used to derive the June forecast. Therefore, the weight
given to early periods is so small that their effect can probably be
disregarded.

The exponential smoothing forecast not only has the advantage of
simplicity, but it is also flexible. The weighting factors may be
changed at any time with no extra computations required. If conditions
seemed to warrant a more stable forecast, one that is less responsive
to random variations, then a weight of only 1/10 may be assigned to
observed demand of the last period and 9/10 assigned to the forecast
for that same period. This combination of weighting tones down the
destabilizing effect of a random variation in the most recent period.
The forecast is further stabilized by assigning more weight to the fore­
cast for the most recent period which in effect gives earlier periods
more influence in the new forecast. As in the case of the weighted
moving average any fractional values may be used as long as the sum of
the two fractions equals one.
CHAPTER III

ORDER QUANTITY

In most inventory situations there tends to be opposing costs. That is, there are costs associated with doing too much and costs associated with doing too little. As the size of the inventory increases, certain costs tend to rise with it. On the other hand, the cost of ordering and related activities such as handling tend to go down if the size of the inventory increases. Apart from the actual capital invested in the stock itself there is the cost of carrying inventory, which includes the operational costs of ordering, storage space, inventory taxes, insurance, and handling charges, to name just a few of the possible costs involved. The inventory manager should understand the relationship between total cost of carrying inventory and order quantity.

It is convenient to think of inventory carrying costs as falling into two categories: those which increase as order size increases or vary directly with order size, and those which vary inversely with order size. Among those costs which vary directly with order quantity are insurance, inventory taxes, storage space, deterioration, and the opportunity cost of the capital invested.\(^7\) The last one may be the interest expense if capital is borrowed or the interest foregone if the inventory capital is taken out of savings or securities. Very often some of

\(^7\) Statt and Miller, *op. cit.*, p. 10.
these costs are not discernable though they may be present. It may be
difficult or impossible, for example, to separate the cost of storage
space from the total value of the building. Depending on the situation,
not all of these costs may be present or there may be many more costs
involved than is implied by the short list given above. Concerning the
type and amount of costs present, it is probably safe to say that no
two inventory situations are exactly alike. Often for a particular
situation, one type of cost assumes a critical importance in determin­
ing order quantities as might be the case for a firm that must borrow
capital that has a high interest expense. Some of the other costs might
seem relatively insignificant when compared to the high interest on the
borrowed capital. Professor Harold Bierman estimates that those costs
which tend to increase with order size are approximately equal to 25
percent of the value of the stock in the inventory on an annual basis.\(^8\)
However, Professor Bierman states that the validity of this "rule of
the thumb" is dependent upon the characteristics of the items in inven­
tory and other factors that may be involved. For example, an inventory
stock of high value items such as jewelry involves different types and
magnitudes of carrying costs than would a stock of building materials.
The jewelry might require a large insurance expense because of the risk
of theft while carrying a stock of building materials involves expen­
sive handling and storing facilities.

The expenses which tend to vary inversely or go down as order size
increases are the cost of paper work associated with ordering and the

\(^8\)Harold Bierman, *Financial and Managerial Accounting* (New York:
unit cost of the product itself. To simplify the discussion, the cost of paper work associated with ordering will simply be referred to as the cost of ordering. As far as the frequency of ordering is decreased by placing larger orders then, over a period of time such as a year for example, the total costs of ordering during that period of time are also decreased. Another reason that ordering costs tend to go down as order size increases stems from the fact that if stock levels are too low, frequent stock shortages necessitate expensive emergency or rush orders in addition to the routine orders. Practitioners often refer to these emergency or rush orders as "back orders." This situation usually occurs between the time that a routine order is initiated and the time that it is finally received. The unit cost of the product may go down if quantity discounts are allowed for large orders.

In order to organize these costs into a meaningful formula or relationship, we will use Figure II on the following page which symbolizes the behavior of the stock level of a particular product carried in inventory. The step-like line starting at point Q on the vertical axis of the graph symbolizes the pattern that the stock level might follow over a period of time. The vertical axis measures the number of units in stock and the horizontal axis measures time. The horizontal increments in the step-like line represents the stock level at any one point in time and the vertical increments represent disbursements. Disbursements are assumed to be in randomly varying quantities and frequency. Although demand is assumed to vary in a random fashion, the figure depicts these variations as comparatively small deviations from the long term average. Also, it is assumed that the general level of
Fig. 2. Typical behavior of stock level for one inventory item over time.
demand changes gradually. In order to simplify the explanation, the
necessity for a buffer stock is ignored. Figure II illustrates the
typical "sawtooth" pattern of a stock level where the abrupt vertical
jump is the replenishment or receipt of an order and the step-like line
is the result of a more or less continuous demand.

Assuming that these costs which increase with order size run about
25 percent on an annual basis of the value of the merchandise in stock,
as is estimated by Professor Bierman, then it follows that the per unit
carrying charge would be about 25 percent of the dollar value of one
unit. For the sake of discussion, those costs which increase with
order size will simply be referred to as the holding cost and those
costs which decrease inversely with order size will be referred to as
ordering costs. The holding cost over the period of one year would be
approximately equal to 25 percent of the average value of the inventory
during that period. If demand were to remain constant during the year
then the average number of units in stock would be given by the area
of ABC in the figure. The shape of the area enclosed by ABC is roughly
that of a triangle. The height of the triangle is equal to Q, the order
quantity. The base of the triangle is equal to one unit of time.
Therefore, by using the formula for the area of a triangle, 1/2 base x
height, the average number of units in stock is equal to 1/2 x Q x 1,
or Q/2. The holding cost then is equal to the average number of units
in stock, Q/2, times the unit cost, C, times percent holding charge or
25 percent. In short: .25CQ/2.

The ordering cost on the other hand is a function of the demand,
order quantity and the cost per individual order. As in the case of
holding cost, the ordering cost is also calculated on an annual basis. Both types of costs may be computed on the basis of any length time period as long as it is the same for each. A year is used here simply because it is probably the most convenient in real situations. The total ordering cost incurred during a year is equal to the cost per order multiplied times the number of orders. The number of orders required during the year may be computed by dividing total estimated demand for the year by order quantity. Where estimated annual demand is denoted by D and order quantity by Q, then the number of orders per year is D/Q. If a is the cost of an individual order then ordering cost for a year is aD/Q. For example, if the estimated demand for the year is 800 units and the cost of an individual order is 2 dollars, then the annual cost of ordering 200 units at a time would be 8 dollars; 800 divided by 200 times 2 dollars. The total variable costs (TVC) involved in carrying inventory for one year is the sum of ordering cost and holding cost. In short:

\[ \text{TVC} = \frac{.25Q}{2} + \frac{aD}{Q} \]

In most inventory situations the TVC curve will be U-shaped as is the one shown in Figure III. The vertical axis in Figure III measures cost and the horizontal axis measures order quantity. The line labeled ordering cost slopes downward, indicating that ordering cost/year declines as order quantity increases. The line labeled holding cost/year slopes upward from the origin, indicating that holding cost varies directly with order quantity. The U-shaped characteristic of the total variable cost curve shows that there is a particular size of order quantity or sometimes a range of sizes for which total variable cost
Fig. 3. Yearly inventory costs when annual demand, cost per unit and cost per order are held constant but order quantity is allowed to vary.

Note: The total variable cost function is flat or has zero slope at the EOQ point therefore the first derivative is equal to zero at that point.

Significant variables:

- $C =$ Cost/unit
- $Q =$ Order quantity
- $D =$ Annual demand
- $a =$ Cost/order
The order quantity which corresponds to the lowest point on the TVC curve is usually referred to as the economic order quantity because it minimizes the total variable costs of carrying inventory. Since the total variable cost curve has zero slope at its lowest point, the order quantity that corresponds to this point may be found by taking the first derivative with respect to Q which is as follows:

First derivative for Q is \( \frac{.25C}{2} - \frac{aD}{Q^2} \)

Then setting it equal to zero and solving for Q gives:

\[ Q = \left( \frac{2aD}{.25C} \right)^{\frac{1}{2}} \]

Theoretically then the formula \( \left( \frac{2aD}{.25C} \right)^{\frac{1}{2}} \) gives the order size which minimizes all or most of the expenses of carrying inventory.

Since it is not always practical for small retail firms to go through the involved process of computing order quantities, the following describes a much simplified method of deriving order quantities recommended by Caterpillar Tractor Company for use by Caterpillar dealers. In principle, the method is probably applicable to a variety of situations. It should be mentioned that the method does not yield an economic order quantity as such for it ignores most of the costs or expenses of carrying inventory which are accounted for by the economic order quantity formula.

In short, the method described by the Caterpillar Tractor Company simply sets the order quantity equal to the forecast demand during lead time plus a 30-day safety margin. For example, if lead time is 60 days then enough stock would be ordered to last for 90 days at the forecasted usage rate. The 30-day safety margin allows for unforeseen variations.
in lead time and usage rate. The lead time plus 30-day safety margin is called the "control period." The usage rate is simply the demand forecast for a 12-month period. If forecasted demand or usage rate for 12 months is 120 units, then a 90-day supply of the item would be 30 units. Ninety days is one-fourth of a year, thus the required stock is $\frac{1}{4}$ of the 12-month demand of 120 units.

Assuming constant demand and lead time, Figure IV on the following page illustrates the behavior of stock levels under the above described method of deriving order quantities. The horizontal axis represents time and the vertical axis represents quantity. As in Figure II, the sawtooth pattern is defined by periodic replenishment resulting in an abrupt rise in stock level and frequent disbursements represented by the sloping portion of the sawtooth. However, in Figure IV the sawtooth line which varies between points b and c on the vertical axis represents quantity on order plus quantity on hand (QOO plus QOH). QOO plus QOH is often referred to as quantity available. The line which varies between a and b on the vertical axis is quantity on hand. As quantity available declines to a point equal to the order quantity derived on the basis of control period as described earlier, a new order is placed. In this case the safety margin is excluded so that the control period is identical to lead time. As a result, stock is replenished at the moment that quantity on hand becomes zero. In the ideal situation quantity available (QOO plus QOH) never drops below order quantity. Consequently, it can be seen from Figure IV that order quantity also serves as the order point. That is, it not only says how much to order but also when to order.
Fig. 4. Idealized picture of a system where the order point, $b$, is also equal to the order quantity, $b_0$. Saw-tooth pattern which varies between $b$ and $c$ represents the behavior of quantity on order and the saw-tooth which varies between $a$ and $b$ represents quantity on hand. Thus order quantity depends upon lead time and demand.
If the assumption of constant demand is relaxed and a safety margin added to lead time then the quantity on order cycle will tend to lag behind the quantity on hand cycle. This is because the safety margin causes replenishment to take place before quantity on hand reaches zero. As a consequence, quantity available reaches the order point at some time after quantity on hand has been replenished. Variations in demand from that forecasted will tend to be absorbed by the safety margin, depending, of course, on how conservative the margin is.

The important thing to remember about this system is that the size of the order depends upon usage rate or demand and lead time or "control period." For a given usage rate, the longer the lead time and safety margin used, the larger the order will be. For example, given a substantial volume of demand, a lead time of six months would probably result in excessively large orders pushing carrying costs out to the extreme end of the total variable cost curve. Conversely, a short lead time of a week or less, for example, would require so many orders to be placed per year that ordering costs might be pushed to the other extreme of the variable cost curve. The only real advantage of this type of method seems to be that it provides an automatic control on stock levels by limiting them to an amount that can be sold during lead time.
CHAPTER IV

ORDER SCHEDULING

Perhaps the most general characteristics of the problem are best illustrated by two basic types of inventory systems defined by Professor John E. Biegel.\textsuperscript{9} One is the fixed order size inventory system, and the other is the fixed order interval inventory system. In the fixed order size type of system the order quantity remains the same or may be predetermined in accordance with achieving various goals such as minimum carrying costs as given by the classical economic order quantity formula that was discussed in the previous chapter. Once an order quantity has been decided upon for placing an order, the time lapse until the next order is placed, i.e., the order interval, is determined by the rate that demand is realized. In other words, the order interval varies as demand varies, given a particular order quantity. If stock is disbursed rapidly to meet a high level of demand, the order interval is shortened. Low demand lengthens the order interval. Thus, under the "fixed order size system" the length of the order interval depends upon the order size and realized rate of demand. When demand for a particular item experiences a certain amount of random variation, then the order interval is constantly changing as a result.

Under the fixed order size system the time to place an order is signaled when stock level reaches an order point. The order point is simply the amount of stock required to meet demand during the time lapse between ordering and receiving the shipment from the factory or distributor. As mentioned previously this delay is usually referred to as lead time. A common method of calculating the order point for a particular item is to convert the demand forecast into a "usage rate per day," then multiply it by the number of days in the lead time. For example, if demand was forecast to be 1095 units for a year, then the usage rate per day would be 3 units. If the lead time was 15 days, then the order point would be 45 units. A more general type of formula is as follows:

\[
\text{Order point} = \frac{\text{Yearly demand}}{365} \times \text{lead time}
\]

Under the fixed order interval inventory system, the order interval remains constant while the order size varies in response to variations in demand, or at least variations in the demand forecast. In this system ordering is scheduled to be conducted at certain predetermined intervals such as a certain day each month for example. The fixed order interval system is one of the simplest methods of controlling inventory; consequently, it is used by many small store owners. In actual situations it is often associated with the practice of taking physical inventory of the merchandise in stock. That is, at an appointed day each month, for example, the manager counts the merchandise on the shelves, making note of shortages and surpluses, then he orders enough of each item to last him through the next month. If he feels that
business is picking up he increases his order quantities accordingly. This method, of course, requires very little in the way of paper work. To be more accurate he will order enough to last through the order interval plus whatever amount is needed to cover lead time.

The fixed order interval system has several important weaknesses. The order interval is often chosen for convenience rather than for economically efficient order sizes. This system also increases the probability of shortages or surpluses because a certain order size is placed to cover a longer period of time. That is, in the fixed order size system, shortages or surpluses only have time to develop during the lead time. In fixed order interval systems, on the other hand, a certain order quantity has been committed to cover demand during both lead time and the interval between orders. Consequently, the fixed order interval system requires a larger safety or buffer stock to protect against shortages than is required to offer the same amount of protection in the fixed order size system.

In a very large retail establishment the fixed order interval system may have a tendency to break down. Taking physical inventory of a large number of merchandise items becomes a very tedious and time consuming task; thus the process may be too slow to keep excessive shortages and surpluses from developing.

The main advantage of the fixed order interval system is its simplicity and convenience. While the order point method of scheduling requires a perpetual inventory system for continually updating stock levels each time a sale or disbursement is made, the fixed order

\[^{10}\text{Ibid.}, \text{p. 77.}\]
interval system on the other hand does not require such constant monitoring of stock levels for orders are placed at the scheduled time regardless of stock levels. In order to get away from the work normally associated with updating or monitoring stock levels required for the order point method, the "two bin" method may be used. In this method there are literally two bins for each item in the inventory. One bin is used to hold the number of units required to meet demand during lead time and the other bin holds the remainder of the stock. Stock is disbursed from the latter bin until it is emptied. When the last unit is removed an order is immediately placed and stock is issued from the first bin until the new order arrives. The bin used during lead time may also contain a safety or buffer stock to allow for variations in demand and lead time. The two bin system offers many of the advantages of the order point method while retaining the simplicity of the fixed order interval type of order scheduling.
CHAPTER V

WESTMONT TRACTOR COMPANY

The Westmont Tractor Company is a distributor of tractors and construction equipment manufactured by the Caterpillar Tractor Company. The firm is the sole distributor of Caterpillar equipment in the Missoula, Montana area. Because of good growth potential in lumbering, agriculture, road construction and other related industries, the Missoula area constitutes a substantial and continuing market for heavy construction equipment manufactured by Caterpillar Tractor Company. The Westmont Tractor Company maintains high quality parts and repair services for units of equipment in the field. A large investment in inventory is required to provide the level of service that customers demand and expect. The following describes the methods and procedures used by Westmont Tractor Company to manage its inventory.

Parts and stock levels are accounted for on a perpetual basis. That is, disbursements and receipts are continually added or subtracted from previous stock levels in order to give an up-to-date picture of stock in inventory. The system has been computerized by tying into a central computer in Peoria, Illinois, with an input device located in the Missoula offices. The computer performs all of the arithmetic involved in updating stock levels and computing order quantities. Each week a huge computer printout is sent back from Peoria to Westmont, which gives a record of various types of transactions that have affected each part in Westmont's inventory. On the following page is a picture.
Fig. 5. Stock record card. The card is used to continually update the status of an item in a perpetual inventory system.
of a stock record card similar to one sheet of computer printout. Stock record cards were used before the system was programmed into the computer. Each sheet of printout contains the transaction history for one part in the inventory.

Along the top line of the printout sheet is given the source of supply of the part, the minimum quantity and the identification number of the part. In the vertical columns the date, type of transaction and results of updating calculations are shown. The total available column represents quantity on hand plus quantity on order. Total sales include all calls for the item whether or not the item was in stock at the time (on hand), or a sale actually made. This way the total demand for the item is recorded. When a sale is made, the number of units is subtracted from the quantity on hand and total available. The amount is recorded in the sales column and added to total sales. The total sales at the end of a quarter is transferred to a quarterly sales history form.

When total available becomes equal to or less than the prescribed minimum, a new order is immediately initiated. If the unit cost of the part is less than 50 dollars, the computer automatically places the order itself. If the unit cost is over 50 dollars, however, the part is listed in another printout along with all other such items ready for order and sent to the inventory manager at Westmont Tractor Company. The inventory manager reviews these items and may make adjustments in light of his own experience and understanding of the demand for that particular item. The minimum, of course, is the amount of stock required to meet demand during lead time.
The minimum is computed on the basis of a "control period." The control period is simply the lead time plus a 30-day safety margin to absorb variations in lead time and forecasted demand. The minimum is calculated by converting the control period into the fraction of a year that it represents and multiplying total sales realized during the previous four quarters by this fraction. For example, if the previous 12 months' sales had been 120 units and the control period is 60 days which is 1/6 of a year, then the minimum is 1/6 of 120 or 20 units. Minimums are refigured quarterly to reflect changes in demand and lead times.

Infrequently sold parts are sold on an order basis, and are accounted for on a nonstock history card. If more than three units are sold yearly, the part is then carried in stock. If a part carried in stock drops below three units per year, then it is discontinued from inventory and sold on an order basis.

Inexpensive parts which move at a rate of twelve units per year or more but have a total consumer's value of 50 dollars or less are ordered on a yearly basis and the minimum is based on a 120-day control period.

Before the inventory system was computerized the rule for determining order quantity was to "order the equivalent of the minimum when the stock available (on hand and on order) reaches the established minimum." The effect of the rule, of course, is that quantity on hand tends to vary between the minimum and zero. The safety margin of 30 days included in the control period tends to create a buffer stock, thereby reducing the chance of quantity on hand actually falling to zero.

Unfortunately, since Westmont's inventory control has been computerized, the writer was unable to learn the details of how order
quantities are determined. According to the general manager of the firm, the data processing service in Peoria, Illinois, applies an economic order quantity formula which takes into account insurance, labor or handling charges, storage, taxes and other carrying costs. The writer can only assume that this formula is similar in principle to the one described in the chapter about order quantities.

The inventory itself is stored in a couple of cavernous storage rooms. One room contains rows of shelves and bins for the storage of smaller items such as nuts, bolts, connecting rods, etc. The other room contains generally larger items such as engine blocks, transmissions which are stored in rows on the floor so that they may be handled by forklifts. Each item is listed in a parts directory which gives a positive identification and description of the item. The parts directory also gives the physical location of the item in the storage rooms. Since the main purpose of the parts directory is simply to tell the physical location of the item, so that it may be found when needed, it is not a control device. That is, it does not include any data related to order quantities, stock levels, etc.

The operational processes of inventory control are subject to several broad policy guidelines. One such guideline is that inventory should turn over at least three times per year. If turnover is less, the top management of the firm will take corrective measures regardless of what formulas and computer say. Since a high level of customer satisfaction is also an important objective to the management of Westmont Tractor Company, they do not hesitate to take the necessary steps needed to hold the number of stock-outs at a minimal level.
CHAPTER VI

CONCLUSION

Although demand is seldom known with certainty, one should be able to conclude that most successful retail firms have been built around certain merchandise items for which demand is to some extent predictable. To invest in merchandise with no idea of demand levels seems to be rather foolish. Consequently, many techniques for predicting demand will provide a close enough approximation of realized demand to make useful inventory decisions. The exponential smoothing method discussed in the second chapter provides the manager of a small firm with a simple and accurate means of forecasting demand from historical records. There are other methods of predicting, such as regression and correlation analysis; however, the increased complexity involved tends to make them impractical for small retail firms. The relatively small gain in accuracy is simply not enough to justify the amount of extra work required.

Since many forecasting methods such as exponential smoothing are reasonably accurate, the classical economic order quantity formula discussed in chapter three may be used in conjunction with these methods even though it is supposed to be based upon the assumption that demand is known with certainty. The writer believes that the main limitations in applying the economic order quantity formula lie in practical considerations rather than theoretical limitations. For example, it would seem that assigning a figure for storage costs is a rather arbitrary
or subjective process in many cases. Isn't the investment in storage facilities usually a "sunk cost" at the time most order size decisions are made and, therefore, probably unaffected by a certain range of order sizes? On the other hand, certain costs such as the interest cost of the money invested, insurance and inventory taxes seem to have a more tangible relationship with order size. Because of their more immediate and direct relationship to order size the writer believes that they may be more relevant in deriving order quantities than storage costs. For example, some managers complain about the amount of money that their inventory soaks up and the difficulty of getting the money. In this case a carrying cost based primarily upon the high interest cost of money invested in the inventory would help protect a precarious working capital position from the danger of insolvency.

Another problem of a practical nature is the amount of arithmetic required to solve the formula \( Q = \left(\frac{2aD}{AC}\right)^{\frac{1}{2}} \) (where \( A \) is the holding rate). In order to be effective, \( Q \) should be recalculated two or three times per year for each item in the inventory in response to changes in demand. Obviously when several hundred items are involved the task becomes too great for the resources of a small firm. In some cases the resultant savings may be enough to justify the hiring of a data processing firm to handle the work. Very often, however, a large percentage of the dollar volume of business is carried on with a relatively small percentage of the total number of merchandise items in the inventory. These high volume items are often referred to as "bread and butter items." The implication is, of course, that the order quantity formula could be applied to a small percentage of merchandise items, yet
constitute effective control over a major portion of the inventory investment.

The arithmetic work load may also be eased in many cases by the use of the "EOQ cost constant." Since the cost per order and the holding rate probably tend to remain constant or at least change very slowly over a long period of time for a particular item, they may be extracted from the formula, \( Q = \left( \frac{2aD}{AC} \right)^{\frac{1}{2}} \), in the form of the constant \( K = \left( \frac{2a}{A} \right)^{\frac{1}{2}} \), in accordance with the rules of algebra. Then each time that order quantities must be recalculated due to changes in demand and price, the simplified form, \( Q = \left( \frac{KD}{C} \right)^{\frac{1}{2}} \) may be used. Only the values for demand, \( D \), and cost, \( C \), change in subsequent recalculation.

In many situations the cost per order and the carrying cost ratio also remain the same from one merchandise item to the next so that the constant \( K = \left( \frac{2a}{A} \right)^{\frac{1}{2}} \) may be the same for a number of different items. This, of course, further reduces the total number of computations required to use the economic order quantity formula.

Deriving order quantities on the basis of lead time, order interval, or "control period" as described in the last part of chapter three, is a rather arbitrary process. That is, such methods in themselves say nothing about what is the most efficient or economic size investment in inventory. An experienced manager may be able to judge or determine from experience what seems to be an economic level of investment and thus manipulate order quantities to achieve that level of investment. The writer feels, however, that the economic order quantity formula is

\[ 11 \] Prichard and Engle, op. cit., p. 63.
a potentially superior control device because it may be made to account for changes in demand, unit cost and other costs which combine to produce a situation outside of the range of experiences of any one person.

Admittedly the techniques discussed in this paper do not cover all of the problems and decisions faced by an inventory manager. The control techniques in this paper are concerned with the operational aspects of inventory control; that is, those activities which are highly repetitive. Many other decisions concerning inventory must be made in light of broad policy objectives held by the managers of a firm. For example, the number of stockouts tolerated must be determined on the basis of management's policies toward customer satisfaction and long run health of the firm. In other cases the number and variety of different merchandise items carried is determined with respect to competition, promotional efforts and customer satisfaction.
A. BOOKS


B. ARTICLES


