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Precision of volume estimates from point sampling for different sampling intensities and procedures

Douglas R. Schnare
The University of Montana

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THE PRECISION OF VOLUME ESTIMATES FROM POINT SAMPLING FOR DIFFERENT SAMPLING INTENSITIES AND PROCEDURES

By

Douglas R. Schnare

B.S., University of Massachusetts, 1977

Presented in partial fulfillment of the requirements for the degree of

Master of Science in Forestry

UNIVERSITY OF MONTANA

1985

Approved by:

[Signatures and dates]
ABSTRACT

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The Precision of Volume Estimates from Point Sampling for Different Sampling Intensities and Procedures (39 pp.)

Director: Dr. Hans Zuuring

Point sampling is the most commonly used method of volume estimation in western Montana. Even though this is true, most forest managers do not understand the effects of different basal area factors (BAF) on the precision of volume estimates. This same misunderstanding is evident when using different subsets of “in” trees for diameter and height measurements or diameter measurements alone. The variances associated with the mean cubic foot volume per acre were calculated for point sampling procedures utilizing four different BAFs and four different subsets of “in” trees. To compare between these point sampling procedures, relative efficiencies were calculated using a point sampling method employing a BAF=20 and a 1:1 correspondence between basal area and the volume to basal area ratio as a standard. Two sets of relative efficiencies were calculated. The first set utilized both diameter and height for predicting individual tree volume, while the second set used only diameter. Separate relative efficiencies were calculated for three stands where tree size and spacing were distinctly different. Results show that the efficiency changes were quite large when changing BAFs while holding everything else constant. Efficiency changes were much smaller when changing between the different subsets of “in” trees. Also the more homogeneous the stand was with respect to spacing and tree size the greater the change in efficiency for the different BAFs. Additionally, the prediction of volume from diameter alone led to a substantial reduction of the variance of the mean cubic foot volume when compared to that found when volume was predicted from both diameter and height.
ACKNOWLEDGEMENTS

I would like to thank Dr. Hans Zuuring for his continued support and assistance throughout this study. Without his help this study would not have been possible. I would also like to thank Dr. Alan McQuillan and Dr. Robert Hollister for serving on my committee and supporting this work with their encouragement and helpful comments. Special thanks must go to John Hoeglund, James Hasbrouck, and Barry Dutton for helping collect the data. Last but not least, I thank my family, Susannah and Benjamin, for the will to carry me through this study.

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CHAPTER ONE

INTRODUCTION

Estimates of the wood volume per unit area are a basic tool of forest management. Foresters routinely use these estimates to decide which activities to perform at any given time. Therefore if the best decisions are going to be made, precise volume estimates are essential.

Volume estimates are determined by using some form of sampling. Many different types and intensities of sampling are available to the forester. In order for the costs of sampling to be held to a minimum with the highest possible precision, an efficient sampling scheme must be chosen for each application.

Point sampling is the most commonly used method of sampling for estimating volumes in Montana. For the sake of cost reduction and consistency, inventory specialists have applied various modifications and rules of thumb to point sampling. For example, the United States Forest Service selects the basal area factor that will produce an average of four to eight "in" trees per sampling point. Other modifications include 1) only measuring heights and diameter breast-heights (DBH) of "in" trees at every second, third or fourth sampling location (this procedure will be referred to as changing the basal area versus volume to basal area ratio (VBAR) correspondence systematically) and 2) measuring DBH on all "in" trees on all sampling points, while only measuring height on a small subset of these "in" trees. Some obvious questions come to mind, for instance do these modifications result in the most efficient method of sampling forest stands in Montana and what is the relative magnitude of the loss in precision of the volume estimates as a result of these practices? These are difficult questions to answer as there is little information available concerning the efficiency of different types and
intensities of sampling. Most foresters do not realize the implications of using different basal area factors. They also do not realize the effect that measuring heights and DBHs on different subsets of “in” trees have on their volume estimates. The objective of this thesis is to address these issues by calculating and comparing the variances of the volume estimates using several types and intensities of point sampling on three different stands. (For explanation of point sampling see Bell and Alexander (1957).)
CHAPTER TWO

LITERATURE REVIEW

There is a large amount of forestry literature concerning the estimation of wood volume per unit area. Very little of this literature examines the effect of using different basal area factors on the precision of volume estimates within a forest stand. There are even fewer that examine the effects on volume estimates of subsampling heights and diameters. For forest situations that exist in western Montana, there are virtually no publications that examine either of these effects.

In selecting an appropriate basal area factor for a given situation most authors recommend using the optimal average method. This method states that the basal area factor chosen for any stand should average a certain number of "in" trees. Once the basal area factor is chosen for a stand it is used for all points within that stand. Dilworth and Bell (1977) recommend that this optimal average should be between four and eight "in" trees, while Beers and Miller (1964) suggest an average of seven "in" trees. Bruce (1961) maintained that an average of three "in" trees is enough. These authors felt that using a basal area factor that averaged more "in" trees than their optimal average would result in large measurement errors, which would cause an unknown bias in the sample. They also felt that averaging fewer "in" trees would result in an unacceptable sampling error. None of these authors provided any statistical tests demonstrating that their optimal averages were better or worse than any other.

A second approach to this optimal average method is known as the constant tally rule. This rule states that the basal area factor will be selected at each point within a stand to get the desired number of "in" trees. The
procedure involves starting with a high basal area factor and reduce it until the desired number of trees is reached. (It must be remembered that basal area factors are discrete values and usually are in multiples of ten, i.e. 10, 20, 30, 40.) This method as practiced, has been shown to have substantial bias (Wensel et al. 1980) and most authors feel this method of basal area estimation should not be used (Schreuder et al. 1981).

The second question, that involves volume estimation using point sampling, is what effect do the various procedures of measuring height and diameter have on the accuracy and precision of the volume estimate? (It must be remembered that tree volumes are never measured but estimated through volume equations using DBH and height or DBH alone. This process introduces another source of error, which is always ignored.) The most accurate method is to measure the diameter and height of all "in" trees. This method is fairly time-consuming because accurately measuring the height of all "in" trees is relatively slow. Therefore, many short-cut procedures have been developed. These generally follow two approaches. With the first approach, the diameter and height are measured on all the "in" trees on a subset of the sampling points (Palley and Horwitz 1961). From these measurements the volume to basal area ratio is calculated and applied to all the points. With the second method, the diameter on all "in" trees and height on only a subset of these trees are measured. Height, volume, or the volume to basal area ratio is then estimated for all the trees, using a regression equation developed from the subset of trees where both height and diameter were measured. Then the volume to basal area ratios are calculated from all the "in" trees. The question here is how many heights should be measured so an accurate estimate of volume can be found. Zeide (1982) in a study using measurement time and sampling error found that approximately one height should be measured for every seven diameter measurements.
CHAPTER THREE

OBJECTIVES

The objective of this study is to examine the effect of three different sampling procedures on the variance of the mean volume per acre estimate for three contrasting forest stands in Missoula County, Montana. I will examine the following procedures.

1- selection of BAF
2- measuring DBHs and heights of "in" trees at every sampling location, every second, third, and fourth sampling location
3- estimating individual tree volume-to-basal area ratio using DBH and height, and DBH alone

The effect of these three procedures on the variance of the mean volume are compared using relative efficiency tables. The standard is the variance associated with the volume estimate using a basal area factor of twenty square feet per acre when all "in" trees are measured for height and DBH.

A fourth objective is to examine the relationship between basal area and the volume-to-basal area ratio when calculating the variance of the mean volume per acre.
CHAPTER FOUR

METHODS AND PROCEDURES

Study Area

The data used in this study were collected from three areas located within twelve miles of Missoula, Montana. The areas were selected subjectively but without bias from forest stands of merchantable size timber. They were chosen to represent distinct forest situations with respect to structure and spacing.

Area one and area three are located five miles east of Missoula in the Pattee canyon drainage. Area one is located on a northwest-facing bench with a slope of ten percent. The stand is mostly eighty-five year old Douglas-fir (72%) and western larch (20%) with some scattered old-growth ponderosa pine (approximately 250 years old). This stand has had a major spruce budworm infestation and the resulting mortality has created lots of openings, thus a heterogeneous spacing pattern. Elevation is 3580 feet.

Area two is located along the Blackfoot river, approximately seven miles east northeast of Bonner, Montana. It is on a level alluvial surface at 3900 feet elevation. The stand is 90 years old with Douglas-fir (81%), lodgepole pine (15%) and ponderosa pine (4%). This stand was chosen because it has a several large openings with some clusters of trees. Stand is generally heterogeneous with respect to spacing, but structure is relatively even-aged.

Area three is located within one-half mile of area one in Pattee canyon. It is on a southeast facing bench with an average slope of 7%. Elevation is 3850 with Douglas-fir (68%) and ponderosa pine (32%). This stand is 85 years old and was thinned to a 12 foot by 12 foot spacing 40 years ago. Since then
some mortality has occurred but spacing is still relatively uniform. Also the
variation in tree diameter and height is less than the other two areas.

Field Procedures

In each area a grid 150 feet on a side was laid out. One corner was
selected as the origin and all trees within the grid were placed on an X, Y
coordinate system. Also all trees outside the grid that would be "in" when a
sampling point was located on the edge of the grid using a basal area factor of
ten square feet were placed on the coordinate system. Trees were located to
the nearest one-half foot interval. All trees less than five inches diameter
breast-height were ignored. Diameter breast-height and total height were
measured on all trees that were on the stem map. Diameter was measured to
the nearest one-tenth (1/10) inch and height was measured to the nearest foot.
The species of all measured trees were also identified. Therefore each tree
had five measured variables; diameter, height, species, X-coordinate, and Y-
coordinate.

Data Handling and Statistical Procedures

All field data were placed on the University of Montana DEC 2065
computer system. Several FORTRAN computer programs were written to
handle calculations, build matrices, and print efficiency tables. SPSSX
procedures were used for the regression of volume on DBH. Each area was
handled as a separate data set. Each tree within a data set was entered as a
separate record. Each record had six fields, one for each of the following
variables: tree number, species, DBH, height, X-coordinate, Y-coordinate. Two
fields were added to each tree record. The first was the the total cubic foot
volume of that tree. This was predicted from the appropriate volume equations
using DBH and height (Faurot 1982). The second field added to each record
was the volume-to-basal-area ratio (VBAR). This was calculated by dividing the
volume by the basal area of that tree (3.14159*(DBH/24)**2).
For each data set, basal area matrices were produced for each of four different basal area factors (10, 20, 30, and 40). Each matrix consisted of elements placed in 76 rows by 76 columns. These corresponded to the stem map produced at each area. Each element represented the number of "in" trees that would be found if a point had been taken at that X,Y location using the appropriate BAF. This is analogous to taking a basal area point at two foot intervals along the grid. Therefore 5776 points existed for each BAF at each area.

Volume to basal area (VBAR) matrices were produced simultaneously with the basal area matrices. There is a one-to-one correspondence between elements of both sets of matrices. Each element of each VBAR matrix represents the average VBAR that would be found for all "in" trees on a point taken at that X,Y location. This is a mean-of-ratios not a ratio-of-means estimator.

The next step was to determine a mean volume per acre and it’s associated variance for the four different BAF’s using all points in the matrices. This was done using the following formulas:

\[
\overline{V} = BAF \times \left( \sum \frac{r_{i,}}{n} \right) / n \quad (1)
\]

\[
s_\overline{V} = BAF \times \left( \sum \frac{r_{i,}^2}{n} \right) - \left( \sum \frac{r_{i,}}{n} \right)^2 / (n-1)n \quad (2)
\]

where:

\[
\overline{V} = \text{mean volume per acre}
\]

BAF = basal area factor
\[ r_{i.} = \sum_{j=1}^{c} r_{ij} \]

\[ r_{ij} = \text{volume/basal area ratio for the } j \text{th "in" tree at the } i \text{th sampling point} \]

\[ c_i = \text{number of "in" trees at the } i \text{th sampling point} \]

\[ n = \text{number of sampling points} \]

\[ 2 \]

\[ s_{\bar{V}} = \text{variance associated with the mean volume per acre estimate} \]

To determine the effect of subsampling on the volume-to basal area ratio, the following procedure was used. Points were taken systematically on the grid. For example, in the case where one-half the points were measured for volume to basal area ratio, every sampling point along the grid was measured for basal area and every other point for volume to basal area ratio. The same procedure was used for every third point and every fourth point. For each of these three sampling schemes, for each basal area factor, a mean volume per acre was calculated along with it's standard error. This was done using the following formulas:

\[ \bar{V} = \text{BAF} \left( \sum_{i=1}^{n} \frac{c_i}{n} \right) \left( \sum_{i=1}^{n} \sum_{j=1}^{c_i} r_{ij} \right) / \sum_{i=1}^{n} c_i \]
\[ s_\_ = BAF \left( \sum_{i=1}^{n_1} r_{i.} - \left( \sum_{j=1}^{n_1} r_{.j} \right) / n_1 \right) / \left( n \left( n_1 - 1 \right) \right) + \]
\[ \sum_{i=1}^{n_1} \left( \sum_{j=1}^{n_1} c_{i \cdot} + r \left( \sum_{j=1}^{n_1} c_{\cdot j} \right) - 2r \left( \sum_{j=1}^{n_1} c_{i \cdot} r_{.j} \right) / \right) \]
\[ \left( n_1 \left( n_1 - 1 \right) \right) * \left( 1 - n_1 / n \right) \]

where:

\[ n_1 = \text{number of sampling points on which VBAR was measured} \]
\[ r = \left( \sum_{i=1}^{n_1} \sum_{j=1}^{n_1} r_{ij} \right) / \left( \sum_{i=1}^{n_1} c_{i \cdot} \right) \]

Once the means and variances were found, the relative efficiency tables were created. Each element within the table is the variance of the control divided by the variance of the method represented by that particular element. The variance of the control was defined as that variance associated with the volume estimate when all sampling points were calculated for VBAR using height and DBH at a BAF of 20 square feet per acre.

Relative efficiency is a commonly used statistic for comparing between different sampling methods. For example, the relative efficiency between stratified and simple random sampling is usually defined as: R.E. =
\[
\frac{1}{s^2_{\text{strat}}} / \left( \frac{1}{s^2_{\text{simp}}} \right) = \frac{s^2_{\text{simp}}}{s^2_{\text{strat}}}. \]

If the relative efficiency is greater than one, stratification of the sample has resulted in a gain in efficiency. Therefore fewer sample points are needed to achieve the same level of precision. If the relative efficiency were less than one, stratification resulted in a loss in efficiency. Then simple random sampling should be used.

For the second set of comparisons, "in" tree volumes were predicted using individual tree basal area. This was done using least squares regression analysis. A separate prediction equation was calculated for each species on each area. All trees of each species were used as long as there were at least 10 trees of that species. If there were not 10 trees for any species on an area then the regression for the species with the most trees was used for that species. Model form was:

\[
\text{tree volume} = b_0 + b_1 \times \text{individual tree basal area}
\]

Once tree volume was predicted, VBAR was calculated by dividing the predicted tree volume by the tree's basal area. The remaining procedures for this set of tests were identical to those for the first set of tests.

Sources of Variation

The sampling error associated with the mean volume per acre for point sampling has two main sources of variation. One is associated with the variation of the basal area estimate, while the second is related to the variation of the volume to basal area ratio. As can be seen from the variance formula (2), these two sources are not totally independent from one another. Therefore examining them separately may not be entirely appropriate, but for simplicity this was done.

For any given situation, the variance associated with the mean basal area per acre estimate is entirely defined by the between-point variation of the number of "in" trees and the BAF (considered a constant). This study will control the BAF by using four different factors on each area. The four selected
will cover the range that are presently used in western Montana. The variation associated with the number of "in" trees was calculated for each BAF on each area.

The sampling error associated with mean volume per acre includes, in addition to the variation mentioned above, the variation of the volume-to-basal area ratio which is somewhat more complicated than that introduced by the basal area estimate. This is primarily due to the way that the volume-to-basal area ratio is determined. The first step in this determination process is to estimate the volume of individual trees from the measurement of DBH and height. Therefore one source of variation is that introduced by this estimation. This study has ignored this source and assumed it would have little effect. The second step is to average the VBAR's at each sampling point. This average is used in the estimate of the sampling error, not the individual tree values. The between-point variation associated with average point VBARs will be completely measured. Also the areas were selected to offer variation between stands with respect to structure. This should result in contrasting conditions with regard to the variation that exists in between-point VBAR's.
CHAPTER SIX

RESULTS AND DISCUSSION

Individual Tree and Stand Characteristics

The first results are the stand and tree characteristics for the three areas sampled. Table 1 lists these characteristics.

Selection of BAF

Relative efficiency tables were created to allow comparison of the variances of the mean volume for the different sampling procedures. The first set of tables were based on sampling procedures that used measured DBH and heights for individual "in" trees. One table was created for each area. The standard for each table is the variance found when all trees are measured for DBH and height using a BAF of 20 square feet per acre. These variance ratios are presented in table 2a, 3a, and 4a. As can be seen from these tables, the relative efficiency of the variance of the mean volume is greatly influenced by the selection of BAF. Area three has the greatest efficiency gain as the BAF changes from 40 to 10 square feet per acre. For the 1:1 correspondence between basal area (BA) and the volume-to-basal area ratio (VBAR) the relative efficiency changed from .45 to 1.84. It is twice the gain in efficiency as area one, which had the least (from .66 to 1.31). For each area the trends were similar for the other BA to VBAR correspondences. The trends within each area are mainly due to the structure of the stands involved. Area three was by far the most uniform with respect to both spacing and tree size (volume). As the average number of "in" trees per sampling location increased (by reducing the BAF) the variance associated with the mean cubic foot volume per acre decreased rapidly. For area one, which had the largest variation in tree size, the variance of the mean decreased much slower. Area two, which had similar
Table 1. Stand and individual tree characteristics for the three study areas.

<table>
<thead>
<tr>
<th></th>
<th>Area one</th>
<th></th>
<th>Area two</th>
<th></th>
<th>Area three</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>s.d.</td>
<td>mean</td>
<td>s.d.</td>
<td>mean</td>
<td>s.d.</td>
</tr>
<tr>
<td>VBAR</td>
<td>26.6</td>
<td>3.5</td>
<td>24.2</td>
<td>4.4</td>
<td>26.1</td>
<td>3.2</td>
</tr>
<tr>
<td>DBH (in.)</td>
<td>10.2</td>
<td>4.2</td>
<td>11.8</td>
<td>4.2</td>
<td>12.4</td>
<td>3.0</td>
</tr>
<tr>
<td>Height (ft.)</td>
<td>66.4</td>
<td>12.4</td>
<td>60.7</td>
<td>13.5</td>
<td>67.8</td>
<td>10.3</td>
</tr>
<tr>
<td>Volume (cu. ft.)</td>
<td>18.7</td>
<td>23.9</td>
<td>22.0</td>
<td>18.7</td>
<td>23.7</td>
<td>13.7</td>
</tr>
<tr>
<td>Basal area (sq. ft.)</td>
<td>.66</td>
<td>.75</td>
<td>.85</td>
<td>.63</td>
<td>.88</td>
<td>.45</td>
</tr>
</tbody>
</table>

Stand characteristics using 20 factor BAF

<table>
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<tr>
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<th></th>
<th>Area two</th>
<th></th>
<th>Area three</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume (cu. ft./ac.)</td>
<td>4111</td>
<td>1424</td>
<td>4071</td>
<td>1426</td>
<td>4077</td>
<td>1020</td>
</tr>
<tr>
<td># of &quot;in&quot; trees</td>
<td>7.3</td>
<td>2.53</td>
<td>8.0</td>
<td>2.82</td>
<td>7.6</td>
<td>1.88</td>
</tr>
<tr>
<td>VBAR</td>
<td>28.1</td>
<td>1.4</td>
<td>25.6</td>
<td>2.0</td>
<td>26.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Table 2a. Relative efficiency table for Area one
when heights and DBHs were measured.

<table>
<thead>
<tr>
<th>BASAL AREA FACTOR (sq. ft./ac.)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA vs. VBAR correspondence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:1</td>
<td>1.31</td>
<td>1.00</td>
<td>.85</td>
<td>.66</td>
</tr>
<tr>
<td>1:2</td>
<td>1.29</td>
<td>.98</td>
<td>.83</td>
<td>.62</td>
</tr>
<tr>
<td>1:3</td>
<td>1.28</td>
<td>.96</td>
<td>.81</td>
<td>.61</td>
</tr>
<tr>
<td>1:4</td>
<td>1.25</td>
<td>.95</td>
<td>.78</td>
<td>.62</td>
</tr>
</tbody>
</table>

Table 2b. Relative efficiency table for Area one
when measuring DBH alone.

<table>
<thead>
<tr>
<th>BASAL AREA FACTOR (sq. ft./ac.)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA vs. VBAR correspondence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:1</td>
<td>1.47</td>
<td>1.13</td>
<td>.93</td>
<td>.71</td>
</tr>
<tr>
<td>1:2</td>
<td>1.45</td>
<td>1.12</td>
<td>.92</td>
<td>.70</td>
</tr>
<tr>
<td>1:3</td>
<td>1.45</td>
<td>1.09</td>
<td>.90</td>
<td>.69</td>
</tr>
<tr>
<td>1:4</td>
<td>1.43</td>
<td>1.09</td>
<td>.86</td>
<td>.68</td>
</tr>
</tbody>
</table>
Table 3a. Relative efficiency table for Area two when measuring heights and DBHs.

<table>
<thead>
<tr>
<th>BASAL AREA FACTOR (sq. ft./ac.)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA vs. VBAR correspondence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:1</td>
<td>1.63</td>
<td>1.00</td>
<td>.70</td>
<td>.52</td>
</tr>
<tr>
<td>1:2</td>
<td>1.57</td>
<td>.97</td>
<td>.67</td>
<td>.50</td>
</tr>
<tr>
<td>1:3</td>
<td>1.52</td>
<td>.93</td>
<td>.64</td>
<td>.48</td>
</tr>
<tr>
<td>1:4</td>
<td>1.44</td>
<td>.89</td>
<td>.62</td>
<td>.47</td>
</tr>
</tbody>
</table>

Table 3b. Relative efficiency table for Area two when measuring DBH alone.

<table>
<thead>
<tr>
<th>BASAL AREA FACTOR (sq. ft./ac.)</th>
<th>10</th>
<th>20</th>
<th>30</th>
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<tbody>
<tr>
<td>BA vs. VBAR correspondence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:1</td>
<td>1.88</td>
<td>1.15</td>
<td>.78</td>
<td>.57</td>
</tr>
<tr>
<td>1:2</td>
<td>1.79</td>
<td>1.12</td>
<td>.77</td>
<td>.55</td>
</tr>
<tr>
<td>1:3</td>
<td>1.75</td>
<td>1.07</td>
<td>.74</td>
<td>.53</td>
</tr>
<tr>
<td>1:4</td>
<td>1.66</td>
<td>1.07</td>
<td>.75</td>
<td>.53</td>
</tr>
</tbody>
</table>
Table 4a. Relative efficiency table for Area three when measuring DBHs and heights.

**BASAL AREA FACTOR (sq. ft./ac.)**

<table>
<thead>
<tr>
<th>BA vs. VBAR correspondence</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>1.84</td>
<td>1.00</td>
<td>.62</td>
<td>.45</td>
</tr>
<tr>
<td>1:2</td>
<td>1.75</td>
<td>.94</td>
<td>.60</td>
<td>.43</td>
</tr>
<tr>
<td>1:3</td>
<td>1.62</td>
<td>.90</td>
<td>.56</td>
<td>.42</td>
</tr>
<tr>
<td>1:4</td>
<td>1.59</td>
<td>.86</td>
<td>.55</td>
<td>.40</td>
</tr>
</tbody>
</table>

Table 4b. Relative efficiency table for Area three when measuring DBH alone.

**BASAL AREA FACTOR (sq. ft./ac.)**

<table>
<thead>
<tr>
<th>BA vs. VBAR correspondence</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>1.91</td>
<td>1.11</td>
<td>.69</td>
<td>.50</td>
</tr>
<tr>
<td>1:2</td>
<td>1.91</td>
<td>1.09</td>
<td>.68</td>
<td>.49</td>
</tr>
<tr>
<td>1:3</td>
<td>1.86</td>
<td>1.08</td>
<td>.67</td>
<td>.49</td>
</tr>
<tr>
<td>1:4</td>
<td>1.91</td>
<td>1.08</td>
<td>.67</td>
<td>.47</td>
</tr>
</tbody>
</table>
spacing as area one but less variation in tree size, was intermediate between area one and three with respect to the relative efficiency gain. It’s relative efficiency for the 1:1 correspondence changed from .52 to 1.63 as the BAF changed from 40 square feet per acre to 10. Within each table, the increase in relative efficiency is greatest when changing from a BAF of 20 square feet per acre to 10 square feet per acre and the smallest is when changing from a 40 to a 30 BAF. For example on area three for the 1:1 correspondence between BA and VBAR the relative efficiency changed from 1.00 to 1.84 as the BAF changed from 20 to 10, but only changed from .45 to .62 as the BAF changed from 40 to 30 square feet per acre. The efficiency gain is approximately twice as much as the BAF changes from a 20 to 10 as it is changing from a BAF of 30 to 20 square feet and twice as much when changing from a BAF of 30 to 20 as it is changing from 40 to 30. For example the relative efficiency on area two using a 1:1 correspondence changes from .52 to .70 to 1.00 to 1.63 as the BAF changes from 40 square feet per acre to 30 to 20 to 10. This indicates that efficiency gains follow a consistent pattern within a given stand as the BAF changes from 40 to 10. The major differences are in the magnitude of the relative efficiency gains, which vary considerably between stands.

**Sampling for Heights and DBHs on Every Point, Every Second Point, Every Third Point, and Every Fourth Point**

The change in the relative efficiency was much less for the different sampling procedures (measuring heights and DBHs on different fractions of the sampling points) for all BAFs used. The relative efficiency decreased in all areas as the correspondence between basal area (BA) and VBAR changed from 1:1 to 1:4. Again area three had the largest relative efficiency change while area one had the least. For example on area three the relative efficiency changed from 1.00 to .86 for the 20 BAF when the correspondence of the BA to VBAR changed from 1:1 to 1:4, while for the same situation for area one the change was from 1.00 to .95. These losses in efficiency are directly related to the variation of the average sampling point VBAR around the overall average VBAR and the size of the variance associated with the sampling method used.
as the standard for that area. Area three had a standard which was about one half that of either of the other two areas. Therefore the same absolute change in the variance of the mean would result in a difference of the relative efficiency of twice as much for area three as in either of the other two areas. If only the absolute changes were observed their magnitude would be directly proportional to the between-point variation of the average sampling point VBARs. Thus the change in relative efficiency for area three would be less than area two and more than area one. Area two's large between-point variation was essentially due to the difference in VBARs between Douglas-fir and lodgepole pine. The average VBAR for Douglas-fir was 23.7 while for lodgepole pine it was 27.7. Thus points having lodgepole pine would have larger than average VBARs while the reverse is true for points with Douglas-fir. It is interesting to note that area one had a larger variance associated with individual tree VBARs than area three, but a lower between-point variation.

**Predicting Volume From DBH and Height, and DBH Alone**

Before the second set of tables were produced, equations that predicted individual tree volume from DBH measurements were generated using simple linear least squares regression, where both an intercept and slope coefficient were estimated. Separate equations were generated for each species on an area that was represented by at least ten trees. Regressions with associated statistics are presented in Table 5. The same standard was used as in the first set of tables so they could be compared (BAF=20 when all points are measured for height and DBH). These results are presented in Table 2b, 3b, and 4b.

The relative efficiencies for the variance of the mean cubic foot volume per acre when predicting volume from DBH alone are greater than all comparable values when volume is predicted from both DBH and height. This is partially caused by a bias in the estimation of individual tree VBAR. When individual tree volumes were predicted from DBH alone, the associated VBARs were underestimated by an average of 2%. Thus the variance of the mean volume was also underestimated. A second cause for the higher relative
Table 5. Regression equations and statistics for volumes predicted from DBH.

<table>
<thead>
<tr>
<th>Area</th>
<th>Species</th>
<th>n</th>
<th>b0</th>
<th>b1</th>
<th>r</th>
<th>standard error of estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area one</td>
<td>ponderosa pine</td>
<td>19</td>
<td>-5.7876</td>
<td>32.782</td>
<td>.99</td>
<td>7.51</td>
</tr>
<tr>
<td></td>
<td>Douglas-fir</td>
<td>187</td>
<td>-1.8538</td>
<td>30.822</td>
<td>.98</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>western larch</td>
<td>42</td>
<td>-1.7612</td>
<td>32.239</td>
<td>.99</td>
<td>1.34</td>
</tr>
<tr>
<td>Area two</td>
<td>Douglas-fir</td>
<td>165</td>
<td>-3.2554</td>
<td>29.192</td>
<td>.97</td>
<td>3.10</td>
</tr>
<tr>
<td></td>
<td>lodgepole pine</td>
<td>26</td>
<td>-1.7809</td>
<td>32.237</td>
<td>.97</td>
<td>1.40</td>
</tr>
<tr>
<td>Area three</td>
<td>ponderosa pine</td>
<td>53</td>
<td>-1.8692</td>
<td>29.729</td>
<td>.96</td>
<td>3.33</td>
</tr>
<tr>
<td></td>
<td>Douglas-fir</td>
<td>114</td>
<td>-2.7675</td>
<td>29.683</td>
<td>.96</td>
<td>1.86</td>
</tr>
</tbody>
</table>
efficiencies for the second set of comparisons is the reduction of between-point variation associated with the VBAR when using volumes predicted from DBH alone. The trend within each table for each area with respect to the effect of BAF selection was similar to the first set of tests. This would be expected as the major effect on the variance of the mean volume estimate is due to the variation associated with the basal area estimate, not the variation associated with the VBAR estimate.

The largest change in the relative efficiency tables when volumes were predicted from DBH and height to when volumes were predicted from DBH alone, was the effect of the different sampling procedures (correspondence between BA and VBAR). Area three had much less efficiency loss when changing from 1:1 to 1:4. Changes in relative efficiency for area one and area two were much less affected. This was essentially caused by the reduction in the variation of the between-point VBAR. For area three this reduction was approximately 50%, while for areas one and two it was approximately 20%. Area two had the largest change in relative efficiency when the BA to VBAR correspondence changed from 1:1 to 1:4. This was due to the fact that the VBARs for lodgepole pine and Douglas-fir were substantially different. Sampling points with lodgepole pine had larger than average VBARs, while the opposite was true for Douglas-fir.

Limitations of Data

The first question to be raised is what effect does sampling at two foot intervals have on the results? Does this sampling procedure introduce bias by way of two-dimensional spatial correlation? This effect was tested using the following procedure. For each area a second set of volume matrices were produced. These consisted of elements placed in six rows by six columns. This was analogous to taking point samples for cubic foot volume per acre at thirty foot intervals. Sampling points taken at these intervals were considered to be independent with respect to neighboring sampling points. The variance of the mean volume per acre was then calculated. These variances were then
compared to the variances found when sampling points were taken two feet apart using the F-test. At the 95% level of confidence all corresponding pairs of variances were not statistically different. This indicates that no bias was introduced by way of two-dimensional spatial correlation.

As only three areas were sampled, an important question is how well does this study represent forest conditions in western Montana. The first item of concern is how well do one-half acre areas represent stands? This is a difficult question to answer directly as there is no published information available that relates the variation of the mean volume per acre of small areas to larger areas. The best way to deal with this subject is to view the basal area estimate separately from the VBAR estimate. Table 6 presents the mean basal area estimate along with the standard deviation of the observations for each BAF used for each area. As can be seen, the standard deviations are relatively large for areas one and two. Based on personal experiences, these standard deviations are larger than average but are by no means extreme for fully stocked merchantable stands in western Montana. Therefore it is the opinion of this author that the one-half acre areas used in this study adequately represent the variation with respect to basal area that would be found on larger areas. Also there is no reason why one-half acre areas (selected because of certain spacing and size characteristics) would not be as variable as larger areas. It could be more likely that these areas would be more variable, with respect to basal area, than stands of similar forest conditions.

As far as the estimate of VBAR and the variation of the observations, it is the same story in that there is no published information to compare with. Therefore the same subjective approach must be used. The largest variation in VBAR was on area two where there were two distinct size classes. About one quarter of the area averaged approximately fifteen feet less in height than the rest. This resulted in greater variation in height and subsequently VBAR than the other areas. Within larger stands (greater than one-half acre) this amount
Table 6. Mean basal area per acre with standard deviation of observations by BAF and area.

<table>
<thead>
<tr>
<th>BAF</th>
<th>Area one</th>
<th></th>
<th>Area two</th>
<th></th>
<th>Area three</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>s.d.</td>
<td>mean</td>
<td>s.d.</td>
<td>mean</td>
<td>s.d.</td>
</tr>
<tr>
<td>10</td>
<td>149</td>
<td>44</td>
<td>159</td>
<td>44</td>
<td>151</td>
<td>29</td>
</tr>
<tr>
<td>20</td>
<td>147</td>
<td>51</td>
<td>160</td>
<td>56</td>
<td>152</td>
<td>38</td>
</tr>
<tr>
<td>30</td>
<td>145</td>
<td>56</td>
<td>161</td>
<td>68</td>
<td>152</td>
<td>47</td>
</tr>
<tr>
<td>40</td>
<td>145</td>
<td>63</td>
<td>161</td>
<td>79</td>
<td>152</td>
<td>55</td>
</tr>
</tbody>
</table>
of variation would not be uncommon. Therefore the variation in VBAR on a small area is probably less than would be found on larger areas. The next question to be asked is what effect does underestimating the variation in VBAR have on the variation of the mean volume. One way of determining this effect is to calculate the variance of the mean volume if VBAR is held constant and compare this to what was found when it was allowed to vary. Table 7 presents the results of this test. As can be seen the variances changed very little when the VBAR was held constant, as a matter of fact the variance actually increased for several situations when the VBAR was held constant. This indicates that even if the between-point variation with respect to the VBAR were doubled the variance of the mean volume would change very slightly.

The second item of concern is which stands in western Montana are the results of this study applicable to? All three areas were selected from areas that had merchantable timber (average DBH greater than 10 inches) and were fully stocked (stand basal areas of 130 square feet or more). Therefore stands having smaller trees or lower stocking could be quite different. It is important when interpreting the results of this study that for any given situation the population of interest should be compared to the study population. If stands are somewhat similar the resulting conclusions will be reasonable.

Relationship Between Basal Area and the Volume-to-Basal Area Ratio When Calculating the Variance of the Mean Cubic Foot Volume

The basal area estimate and the VBAR estimate are not independent when calculating the mean volume and it’s variance. This can be seen from formulas one and two. A good question at this point is, how are they related and does it change when using different BAFs. An easy way of viewing this is to construct a simple linear regression equation with the number of “in” trees as the independent variable and the sum of the individual tree VBARs at the sampling points the dependent variable. This was done for each of the four BAFs used for each area (table 8). If basal area and VBAR were independent one would expect the slope of the regression line to be approximately the
Table 7. A comparison of the variance of the mean volume per acre when individual tree VBARs (IT) are used versus using only the average VBAR (A) by BAF and area.

<table>
<thead>
<tr>
<th>BAF</th>
<th>Area one</th>
<th></th>
<th>Area two</th>
<th></th>
<th>Area three</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IT</td>
<td>A</td>
<td>IT</td>
<td>A</td>
<td>IT</td>
<td>A</td>
</tr>
<tr>
<td>10</td>
<td>268</td>
<td>268</td>
<td>216</td>
<td>220</td>
<td>98</td>
<td>105</td>
</tr>
<tr>
<td>20</td>
<td>351</td>
<td>350</td>
<td>352</td>
<td>355</td>
<td>180</td>
<td>176</td>
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<tr>
<td>30</td>
<td>415</td>
<td>422</td>
<td>504</td>
<td>514</td>
<td>289</td>
<td>277</td>
</tr>
<tr>
<td>40</td>
<td>532</td>
<td>545</td>
<td>674</td>
<td>691</td>
<td>397</td>
<td>381</td>
</tr>
</tbody>
</table>
Table 8. The slope of the regression line using the sum of the VBARs at a sampling point as the dependent variable and the number of "in" trees as the independent variable by BAF and area.

<table>
<thead>
<tr>
<th>BAF</th>
<th>Area one</th>
<th>Area two</th>
<th>Area three</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>27.9</td>
<td>24.5</td>
<td>25.1</td>
</tr>
<tr>
<td>20</td>
<td>27.8</td>
<td>24.5</td>
<td>26.3</td>
</tr>
<tr>
<td>30</td>
<td>27.4</td>
<td>24.5</td>
<td>26.7</td>
</tr>
<tr>
<td>40</td>
<td>27.4</td>
<td>24.5</td>
<td>26.8</td>
</tr>
<tr>
<td>mean point</td>
<td>28.1</td>
<td>25.5</td>
<td>26.8</td>
</tr>
</tbody>
</table>
average point VBAR. As can be seen the slope of the line is generally less than the average point VBAR. Each area shows a different trend. In area three the slope decreases as the BAF changes from 40 to 10. This indicates that with a BAF of 10 the sampling points having a greater number of “in” trees also have an average VBAR that is greater than points with fewer “in” trees. On area one the opposite effect is noted. Using a 40 BAF, sampling points with a greater number of “in” trees have a lower average sampling point VBAR than sampling points with fewer “in” trees. For area two the slope is constant across all BAFs, but it is always less than the average sampling point VBAR. There is a constant trend for sampling points with more “in” trees to have a lower average VBAR. As VBAR is generally related to the DBH of individual trees, this suggests that sampling points with a greater number of “in” trees also have smaller DBHs than sampling points with fewer “in” trees.

Different Methods of Predicting Volume-to-Basal Area Ratios

In most applications of volume estimation using point sampling, heights are measured on only a small proportion of the trees on the variable radius plots. DBH is usually measured on all the “in” trees. Therefore the VBAR on trees where height is not measured must be estimated from DBH alone. The most common procedure is to develop a regression equation from the measured DBH–height pairs and then predict height, volume or VBAR. If height or volume were predicted then VBAR must be calculated from the predicted values. Another common procedure is to calculate the mean VBAR for all trees with measured DBH and heights, and use this mean for all trees. First let’s take a closer look at the use of the prediction equations. This study predicted volume from DBH. This prediction process caused a slight underestimate of the mean cubic foot volume per acre. This bias ranged from 30 cubic feet on area three to 60 cubic feet on area two and amounted to 1 1/2% at the worst. However this process underestimates the variance of the mean volume by an average of 10%. There are two reasons for this. The first is a slight bias in the VBAR prediction by diameter class where the smaller DBHs are underestimated
for VBAR and the larger DBHs are overestimated. The second reason is that the between-point variation is reduced. This coupled with the fact that on points with a greater number of trees the diameters are generally smaller than average and visa versa. The degree with which this occurs varies from stand to stand and is also somewhat dependent on the BAF used.

Now let's investigate how predicting VBAR from DBH, instead of volume from DBH, would effect the estimates of the mean volume and it's variance. Predicting VBAR directly from DBH (or the basal area of individual trees) is somewhat different than the previously described prediction process. This is due to the relationship between DBH and VBAR. It is not a straight line, but curvilinear. There is also a much lower correlation between DBH and VBAR than between DBH and volume. Therefore if one hypothesized an incorrect model form (which would be easy), a large bias would be introduced into both the mean volume estimate and the it's variance. Even if the correct form were used, the large variation of VBAR at all levels of DBH could lead to a substantial error in the variance estimate. This, in turn, would lead to an error in the confidence interval for the mean volume estimate. Therefore it would be impossible to get a true idea of the accuracy of the mean estimate.

How about predicting height from the DBH and then calculating VBAR? This method has the same problems as discussed with the prediction of VBAR, in that the model form is not simple and there is a large variation in height for all levels of DBH. Also there is heterogeneous variance (increasing variation in height with increasing DBH). In addition there is an extra step, that of calculating volume from DBH and the predicted height. Therefore this method is probably the least desirable as further error would enter into the tree volume calculations.

The last method to be discussed is calculating an average VBAR from the measured trees and using it for all trees. This method would ignore the fact that VBAR is correlated to DBH, which would introduce bias into both the mean and it's variance. The variance would have a positive bias (or in other words
the estimate of the variance would be larger than it should be). The magnitude of the bias would depend on the stand conditions encountered.
In western Montana, forest managers often use rules of thumb to determine which BAF to use in any given situation. The most common states that an average of four to eight “in” trees per sampling point should be obtained for a reasonable estimate of the mean per acre volume. Figure 1 shows the variation of the mean cubic foot volume per acre estimate over the average number of “in” trees per point for each of the three areas sampled. As can be seen from the figure there is a large change in the variance of the mean volume as the average number of “in” trees changes from four to eight. The change in the variance is much less as the average changes from eight to twelve. This indicates that the accuracy of the mean volume estimate would be greatly increased for the same number of sampling points if an average of eight “in” trees was obtained instead of four. Therefore it is the opinion of this author that an average of at least eight “in” trees per sampling point should be obtained when sampling for volume. This would maximize the information returned from the survey while holding the costs to a reasonable level.

A second recommendation is that a 1:4 basal area to volume-to-basal area correspondence should be used on all volume surveys. This study found that the precision of the volume estimates were only slightly reduced when the “in” trees were measured for height and DBH on every fourth sampling point as compared to every point. By measuring only one-fourth the “in” trees for height and DBH, time and money could be saved. Alternatively more sampling points could be established for the same cost, thereby increasing the precision. If this method is employed, heights should be measured on all trees where DBH is measured. Otherwise an unknown bias would be introduced into the estimation of the variance of the mean volume.
Figure 1. Variance of the mean volume by the average number of "ln" trees.

Variance of mean volume (cubic feet)

Area 3
Area 2
Area 1

Average number of "ln" trees per sampling point
A third recommendation is that individual tree volumes should be estimated from both DBH and height, and not DBH alone. This study found that when DBH alone was used to predict tree volume, which was derived from a volume equation based on DBH and height, the variance of the mean cubic foot volume per acre was underestimated by 10%. This in turn would cause a substantial reduction in the confidence interval about the mean and mean volume per acre estimates would appear to be more precise than they really are.
LITERATURE CITED


