Differences in empirically measured compensating and equivalent variation measures of consumer's surplus: A case study

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DIFFERENCES IN EMPIRICALLY MEASURED COMPENSATING
AND EQUIVALENT VARIATION MEASURES OF CONSUMER'S
SURPLUS: A CASE STUDY

by

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The implication of neoclassical economic theory is that equivalent and compensating variation values should be nearly equal. However, empirical studies have derived results which are contrary to economic theory. Equivalent and compensating variation have been found to differ by a considerable amount, sometimes as much as two thousand percent.

This paper provides insight into why compensating and equivalent variation values differ when empirically measured. In a contingent valuation scenario, a questionnaire was designed to elicit compensating and equivalent variation values for Missoula, Montana. The three major results of this study are: 1) the calculation of compensating and equivalent variation values for Rattlesnake Creek water, 2) the rejection of the invariance hypothesis, which underlies the most commonly used decision theory, and 3) the presentation of a new decision theory called prospect theory.

The calculation of compensating and equivalent variation values for Rattlesnake Creek drinking water resulted in dollar figures of $2.80 and $6.40, respectively. The magnitude of disparity between these values is similar to many other empirical studies.

Two tests were used in analyzing the validity of the invariance hypothesis. Both of these tests showed that this hypothesis does correctly predict the way in which individuals make decisions. This study shows that individuals' decisions are not symmetric when they are confronted with choices that involve outcomes of gains and losses. For example, it was found that individuals viewed the loss of the right to consume Rattlesnake Creek water much more adversely than they viewed the gain of the right to consume this water. This discrepancy is termed the loss aversion effect.

The final area of interest for this paper concerns prospect theory. It is presented as an alternative decision theory. Prospect theory explicitly assumes that individuals view potential losses much more strongly than they do potential gains. It is assumed in this paper that individuals equate equivalent variation decisions with a loss and compensating variation decisions with a gain. It is proposed that the loss aversion effect plays a substantial role in explaining the empirically measured difference between compensating and equivalent variation.

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Overview

A characteristic of public goods is the inability of the market to put a price upon them. Contingent valuation is a technique which has been developed to alleviate this market failure problem. The technique of contingent valuation creates an artificial market. Within this artificial market, dollar values are derived for the goods under consideration. One method of contingent valuation involves using a questionnaire which can elicit both compensating and equivalent variation values. These two measures of value are used to calculate consumer's surplus. In many studies which use the contingent valuation technique, the compensating and equivalent variation values have differed by a considerable amount.

The focus of this paper is directed toward the two measures of consumer's surplus, compensating and equivalent variation. According to neoclassical economic theory, the measurement of compensating and equivalent variation should result in equivalent values, except for the negligible difference which may accrue due to income effects. Due to the type of good which is usually measured, the income effect is usually
small. However, these two measures of consumer's surplus often differ when empirically measured. Examples of the types of goods which have been measured are: fishing sites, clean air, and free flowing rivers. The following chapters are concerned with why the empirically measured disparity between compensating and equivalent variation occurs.

Outline of Paper

This paper is comprised of five chapters. Chapter one is the introduction for the paper. In chapter two there is a literature review for the concepts of consumer's surplus and risk analysis. Within this chapter, the theoretical concepts are presented, along with the theoretical and empirical problems which have been encountered. Also included within this section is a statement of the problem that this paper addresses.

The literature review relating to risk analysis pertains to the way individuals make decisions under uncertainty. A chronological synopsis of the evolution of risk analysis is given. This synopsis includes mathematical expectations, expected utility, and von Neumann and Morgenstern's theories. A new theory of explaining decisions under risk, referred to as prospect theory, is also presented. The possibility that prospect theory alleviates some of the discrepancy between the compensating and equivalent variation measures is discussed in the conclusion of this chapter.
The proposition that the failure of the hypothesis of invariance leads to the empirically measured discrepancy between consumer's surplus values is presented in chapter three. This hypothesis is part of a set of hypotheses which make up decision theory. It is suggested that invariance fails because of two reasons: nonlinear decision weights and framing bias. To test these hypotheses, an empirical study is designed to see if nonlinear decision weights and framing bias exist in a situation where compensating and equivalent variation are measured. Rattlesnake Creek's drinking water is the commodity which is chosen to elicit these consumer's surplus values.

Chapters four and five focus on the statistical analysis which is derived from the empirical study. There are three main objectives of this analysis. Two objectives are concerned with finding evidence of nonlinear decision weights and framing bias while deriving compensating and equivalent variation values. The third objective is to analyze the variables which have an impact on the two measures of consumer's surplus. The results and their implications are discussed in chapter five.
Chapter II

LITERATURE REVIEW OF CONSUMER'S SURPLUS AND RISK ANALYSIS

2.1 Introduction

The origin of consumer's surplus dates back to the mid 19th century and to the economist Jules Dupuit [Dupuit 1849]. Since its inception, the concept of consumer's surplus has been consistently beset with problems. Curiously, with all of its problems, few economic theorists have given it much attention. This chapter provides the reader with a literature review of consumer's surplus and risk analysis. Section two explains the concept of consumer's surplus as well as some of the different types of consumer's surplus which exist. Section three presents the problem which this paper addresses. This section shows that in empirical tests, two different measures of consumer's surplus result in vastly differing values even though economic theory stipulates that these values be nearly equivalent. The fourth section of this chapter provides a literature review of risk or behaviour under uncertainty. This literature review covers the historical development of risk analysis up to the current state of the art. The fifth and final section of this chapter presents a new theory of risk called prospect theory. Prospect theory appears to have potential for explaining at least some of the discrepancy in measurement between the
two theoretically equivalent measures of consumer's surplus. This potential which prospect theory has for explaining the discrepancy is also explored in this section.

2.2 Definitions of Consumer's Surplus

Alfred Marshall defined consumer's surplus to be: "The excess of the price which he, (the consumer) would be willing to pay rather than go without the thing, over that which he actually does pay" [Marshall 1922 p.124]. However, his hypothesis that the marginal utility of money is constant was found to be incorrect. The clarification of the concepts associated with consumer's surplus can be traced back to the work of Sir John R. Hicks. For an expanded exposition of the theory of consumer's surplus see appendix three.

An income compensated demand curve underlies Hicks' definition of consumer's surplus. For his definition he states: "It is the compensating variation in income, whose loss would just offset the fall in price, and leave the consumer no better off than before" [Hicks 1942 pp.40-41]. This is the area PABP1 in figure 2-1. It is expressed in monetary terms for a price decrease on a Hicksian income compensated demand curve.

Hicks referred to this consumer's surplus as compensating variation. It shows, for a price decrease, how much the consumer would be willing to pay in order to consume at the new price level. The amount that the consumer is willing to pay just erases the gain in real
income which the consumer attains due to the This compensating variation measure can be divided into two separate components. The first segment, PP1AC, is the cost savings which accrues to the consumer on the quantity which he was purchasing before the price change. This rectangle shows the change in what the consumer has to pay. The second component shows the gain which accrues to the consumer due to the increased level of consumption. The points of the hypotenuse of the triangle ABC are a measure of the marginal valuations which the consumer places upon each new unit of output that is consumed. When this marginal valuation is equal to the new price, the consumer will have no incentive to move from that point. Thus, this second segment of consumer's surplus is the difference between the marginal valuations of the extra units AB and the price which is paid for them.

Equivalent Variation

The compensating variation is not the only measure of consumer's surplus to be considered. Henderson pointed out: "The compensatory variation may be different according as to whether we consider the loss of income which is worth bearing in order to obtain the availability of X or alternatively, the gain in income which will compensate for abandoning all purchase of X" [Henderson 1941 p.119].

Hicks referred to this new element as equivalent variation. It can be thought of as the sum of money which changes hands if the consumer is exempted from the price change. This sum of money makes him as well off as if he participated in the new price.
FIGURE 2-1
Compensating Variation
Both compensating and equivalent variation can be perceived from the gain or loss perspective. It depends upon the circumstances. Equivalent variation always measures the amount of money an individual must receive or pay out to keep him at the status quo or as well off as before the price change, the introduction of a new good, etc. Thus, depending upon whether the price of a good increases or decreases, the individual will make payments or receive payments, respectively. Conversely, compensating variation always measures the sum of money that an individual is willing to pay or receive in order to consume at the new price, etc. Thus, for compensating variation, if the price of a good decreases the consumer is willing to pay to be able to consume the good at the lower price.

Equivalent and compensating variation, along with Marshall's definition of consumer's surplus, constitute the most frequently referred to and used measures. There are further measures of consumer's surplus once the constraint of constant utility of money is lifted. But, because compensating and equivalent variation are the most common measures of value and the focus of the latter chapters of this paper, they are the only ones to which I will refer.

Differences in the Measures

It is important to discern the differences between Marshall's and Hicks' measures of consumer's surplus. The first point where they differ is that Marshall's analysis implies a constraint on the quantity purchased. "Marshall's definition corresponds to the amount which the
consumer would be willing to pay, if he could not get any of the commodity otherwise, for the opportunity to buy, at the existing price the amount which he is in fact buying" [Henderson 1941 p.117]. In either of Hicks' definitions, there is no constraint upon the amount of output which the consumer wishes to purchase.

An effective technique that illustrates the magnitudes of difference between the measures involves the use of indifference curves. In figure 2-2, the vertical axis measures income, with M being the consumer's total income, or the expenditure on the n-1 goods. On the horizontal axis lies the n'th good X. The price of X is given by the slope of the budget line ML.

Prior to the introduction of X the consumer has a utility level that corresponds to the indifference curve I (this example follows Mishan 1976 pp.417-418). The introduction of X to the consumer's commodity bundle raises his welfare to I1. His utility is maximized at the tangency point Q1, consuming P amount of X. The three measures of welfare that arise from the introduction of this good can be calculated in the following manner. In this example, the compensating variation measures the maximum amount that the consumer is willing to pay to be able to purchase X at the stated price. The dashed budget line Y1Z is parallel to ML, meaning that income has been decreased but prices have remained constant. The income MY1 is the maximum amount of money that the consumer would pay in order to consume P of X. If the consumer would pay a larger amount, he would be on an indifference curve lower than I, meaning that he is less well off in his new position than he was.
before the introduction of X. MY1 is the amount of money that makes the consumer indifferent to position I where no X is consumed and I1 where OP of X is consumed.

Equivalent variation measures the minimum sum that the consumer would accept to forego any consumption of X. This amount is $Y_2M$. With this sum, his total income level has been elevated enough to place him on the higher indifference curve I1, which is where he would have been if he would have consumed X in the amount that he wished.

In this diagram the Marshallian measure is the vertical distance Q1R. This is the maximum sum the consumer would pay to buy X, provided that he is constrained to buy OP of X. OP is the amount which would be purchased if the consumer had no constraints placed upon him.

Significance of Income Effects

An assumption of Marshalls which affected his analysis of consumer's surplus was that the marginal utility of money was constant. Notice the implications of this assumption. If you assume this to be correct, the slopes of the indifference curves I and I1 at Q1 and R in figure 2-2 would be equal. That is, differences in income would not affect the marginal rate of substitution of X for income. If the slopes of the indifference curves I and I1 are equal, this implies that the vertical distance between the curves is constant. If the marginal utility of money was constant throughout the indifference curves, then the vertical distance between the curves would also be constant throughout. If the vertical distance between the curves was constant
throughout, the amount of money the consumer is willing to pay to consume X, the amount of money the consumer must be paid to forego the consumption of X, and the Marshallian measure of constrained consumption would all be equivalent. If the marginal utility of money is constant it would eliminate any distinction between the three different measures of consumer's surplus.

Since the marginal utility of money is not constant, notice the differing monetary measurements of equivalent and compensating variation. None of the early authors who dealt with the two measures of variation put any constraints on their size. However, they all, in some form, implied that they were equal except for the income effect. Hicks states that Marshall's analysis is a good approximation for his measures so long as income effects are negligible [Hicks 1956]. He defines the magnitude of the change as a relation to the substitution effect. He states: "The income effect must clearly .... be small in relation to the substitution effect itself" [Hicks 1956 p.77]. Hotelling wrote that the only time these measures of value break down is when "the variations under consideration are too large a part of the total economy of the person" [Hotelling 1938 p.289]. Regarding this subject Henderson stated that the level of utility is the same, although "its expression in terms of money income is different since the significance of money varies" [Henderson 1941 p.119]. Further on in his article he concludes that the measure of consumer's surplus that is used is of little import: "since we shall normally expect the results to lie so close together that it would not matter which we choose" [Henderson 1941 p.121].
FIGURE 2-2
Two Measures of Consumer’s Surplus
The concept of compensating and equivalent variation being approximately equal, given the good being measured comprises a small amount of the total budget expenditure, is illustrated below in figure 2-3. The horizontal axis measures the good X. X is a normal commodity with a positive marginal utility of money. Intuitively, the argument follows this path: As the price of X increases relative to the other consumptive goods, the consumer becomes less willing to pay to be able to consume it. Also, the consumer does not require as much compensation to forego the use of X because at the higher price of X, the individual is consuming very little. As the amount of expenditure on the good decreases, both measures of value tend toward zero and each other.

This may be shown graphically. In figure 2-3, the vertical axis measures the composite good Y. Prior to the introduction of good X the individual's level of welfare is given by I. With the introduction of good X the individual's welfare rises to II. The price of the commodity is given by the slope of the budget line MZ. The consumer is maximizing his welfare at the tangency point A. LM and MN measure equivalent and compensating variation values, respectively. It is obvious that equivalent variation is greater than compensating variation. Equally obvious is that the ratio of equivalent variation to compensating variation is greater than one.

Now, assume the price of X increases. Since commodity X is normal, the consumer will consume less of it. If the price continues to increase, the result will be a decrease in the total expenditure on the good X. The new budget line is MU. The higher price of X lowers the
consumer's welfare to I2. The higher price decreases both the compensating and equivalent variation values. These are MP and OM, respectively. Note that both values are decreasing toward zero. As the price of X continues to increase, the welfare level will continue to decrease and approach the original level of utility, I. When the level of welfare descends back to I, the point of tangency between the budget line and the indifference curve is at M, indicating zero consumption of X. It also indicates zero expenditure on good X. It is assumed for this proof that the marginal rate of substitution of X for income is less than or equal to the price ratio of good X to income at the intersection of the budget line and the vertical axis [Leftwich 1979 p.92]. This enables a corner solution to exist. The decreasing of the welfare level results in both the compensating and equivalent variation values approaching closer and closer to zero until the level of utility reaches I. At the utility level signified by the indifference curve I, both measures are zero. However, as these measures approach zero the ratio of equivalent variation to compensating variation approaches one, meaning that these values are nearly equal. This must occur because both values become zero at the same point, where there is no consumption of X. This is true so long as the indifference curves are continuous, which they are by definition.

Recently, many economists have relied on the vague statements of their predecessors in regard to the part income effects play in the discrepancy in measurement between compensating and equivalent variation. For example Freeman states: "No decisive case can be made
for either measure. But, .... in practice this may not be an important issue since the two measures, equivalent and compensating variation will usually not differ significantly" [Freeman 1979 p.33]. The consensus has been that the difference between the two measures would be quite small, depending upon what significance the good has on our total expenditure function.

In Willig [1976] boundaries are calculated for the size of the difference between the two measures for a single price change. Willig shows that in most cases, the error in substituting Marshall's consumer's surplus for Hicks' measures will be less than the errors involved in estimating the income conditions for his constraints. His constraints reflect the worries that were noted above i.e., the good does not take up a large percentage of expendable income.

2.3 Statement of Problem: Empirical Discrepancies

If all of the above statements hold true, is there a problem concerning the difference in measurement between equivalent and compensating variation? Unfortunately, as is often the case, this theory and its empirical applications are in conflict.

In recent years economists have been attempting to place a dollar figure on commodities such as wilderness areas or recreation sites. Economists usually value goods and services based upon market observations. From this data base a demand curve can be constructed and the consumer's surplus can then be derived. The valuation of goods such
as those mentioned above poses a problem due to the resource not being priced, or only partially priced, in a market.

There have been several techniques developed which attempt to simulate an actual market system. These techniques all fall under the heading of contingent valuation. To elicit the consumer's willingness to pay, there are several available options. One of these is the interview method [Knetsch and Davis 1966]. An interview method creates a hypothetical market where prices can be imputed, without any suggestion or example of an existing operating market [Schulze et al. 1981].

The interviewer asks the respondents the maximum amount that they would pay in order to retain the ability to consume a particular good. They are also asked what is the minimum amount of money they would accept in order that they would forego the consumption of the good. These questions, respectively, measure the consumer's compensating and equivalent variation.

It has been asserted that these two measures of value should be nearly equal, with the given income constraints. Unfortunately, the literature is full of studies which show a large disparity between compensating and equivalent variation.
FIGURE 2-3
Income Effects on
Compensating and Equivalent Variation Value
One of the earliest studies using this technique was for the valuation of waterfowl [Hammack and Brown 1974]. Data was gathered through a questionnaire which was mailed to a sample of hunters within the Pacific flyway. Their measure of equivalent variation was $1,044.00 per year, compared to $247.00 per year for compensating variation. They differed by over four hundred percent.

A study by Schulze et al., [1981] imputed valuations by elk hunters in Wyoming. They asked the hunters questions through personal interviews. The average willingness to pay was $54.00 per year to increase sightings of elk from one to five per day. The average willingness to sell the right to see five elk per day was $142.00 [Schulze et al., 1981].

There are numerous other examples that show this disparity (see table 2-1). As previously noted, the equivalent variation value does not have to be greater than the compensating value. In the examples below the equivalent variation happens to be associated with a gain while the compensating variation value is associated with a loss.

It seems highly improbable in the cases cited above and other studies as well, that the cause for the discrepancies is due to the good being a large part of their expenditures relative to their income. In most empirical analyses the goods in question do have an income elasticity that is greater than zero, but this does not explain such a large difference in the measures, especially in aggregation. In an analysis involving income effects, Gordon and Knetsch [1979] found
through regression results that there was no foundation for the income effect being the sole explanation for the differences in the two measures.

Many people believe that the disparity problem lies within the interview procedure and not with the measurements or theory itself [Dwyer and Bowes 1978 and Brookshire et al. 1980]. One problem in using interviews may be hypothetical bias. This would occur when respondents react inaccurately to the seeming irrelevance of the questions. To circumvent this problem the use of a payment vehicle has been tried [Rowe 1980]. Instead of asking for a sum of money which the respondent would either pay or receive, a more plausible situation such as paying higher or lower taxes or utility bills is introduced. This creates a situation to which the consumer can more easily relate.

Table 2-1 is an illustration of the disparity between compensating and equivalent variation values. These studies all involved some method of the contingent valuation format [Meyer 1979 p.226].

In a study designed to totally eliminate hypothetical bias, people were confronted with actual money payments and cash compensations [Knetsch and Sinden 1984]. Over a series of experiments, the compensation that was demanded was approximately four times the value measured by the maximum willingness to pay to attain the same right.
Table 2-1: Studies Showing the Disparity Between Compensating and Equivalent Variation

<table>
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<tr>
<th>Study</th>
<th>Author</th>
<th>Willingness to Pay</th>
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<tbody>
<tr>
<td>Saltwater Recreation</td>
<td>Meyer</td>
<td>19:1</td>
</tr>
<tr>
<td>Favorite Fishing Site</td>
<td>Sinclair</td>
<td>20:1</td>
</tr>
<tr>
<td>Elk Hunting</td>
<td>Brookshire and Randall</td>
<td>7:1</td>
</tr>
<tr>
<td>Wetland Hunting</td>
<td>Hammack and Brown</td>
<td>4:1</td>
</tr>
<tr>
<td>Fishing a Park</td>
<td>Eby</td>
<td>3.5:1</td>
</tr>
<tr>
<td>Local Postal Service</td>
<td>Banford</td>
<td>4:1</td>
</tr>
<tr>
<td>Fishing Pier</td>
<td>Banford</td>
<td>3:1</td>
</tr>
</tbody>
</table>

Many economists put forth the idea that only one of the measures of value is valid, depending upon who possesses the property right [Krutilla and Fisher 1975]. If an individual owns the entitlement, then the proper measure would be to ask the person the minimum amount that he would have to be paid in order for him to forego the right to use the good. According to this hypothesis, if there is a difference between empirically measured compensating and equivalent variation values, then it matters which measure we use and we should use the measure which matches the "appropriate" entitlement. However, if the disparity
between measures is rectified by employing the appropriate entitlement, the validity of the Coase Theorem is questioned [Knetsch 1984]. This theorem postulates that the same efficient allocations of resources are predicted to occur regardless of the initial assignments of rights [Coase 1960].

Other possible problems pertaining to the interview method are strategic, instrument, information, and sampling or interviewer bias. Strategic bias can occur when respondents attempt to influence the study by either over or under bidding their true preferences. To counteract this, several studies have suggested an outlier range of approximately ten standard deviations from the mean bid. Bids outside of this are disregarded. Instrument bias comes from the process which is employed to discover individuals' preferences. For example, if an iterative bidding scheme is used, the dollar amount of the starting bids and the amount and type of information given about the particular good influences the revealed bids. Most studies which now use an iterative bidding system vary the starting level of bids as well as the amount of information. This serves to identify the two biases if they are present in the study. The biases which are associated with sampling and interviewing problems can be limited through careful application of statistical techniques and objective interviewing procedures.

Two other reasons for the disparity in measures which have been mentioned in the literature are: when the question asked implicitly violates individuals perceptions of the appropriate entitlement, you may get "inaccurate" or "emotionally" biased results; and monetary income
may not be the major determinant of individual welfare.

Studies have been done which specifically test for the presence of the aforementioned problems [Schulze et al. 1981]. They were found to the most part to be relatively insignificant. Regardless of whether biases exist within a study, this paper stands by the hypothesis that the biases are consistent across the two measures. Even if there are instances of some types of bias and/or income effects that affect one measure of value differently from the other, they could not completely account for the total disparity between the two measures of value.

2.4 Development of Risk Analysis

When an individual is confronted with a decision without full information, there is a possibility that an error will be made. While the possibility of an error may be either large or small, the lack of complete information results in some risk or uncertainty to the decision maker. This section presents the different theories that have evolved concerning the way people behave and make decisions under uncertainty.

From the earliest analysis, gambling for a monetary reward has provided an excellent example of decision making under uncertainty. In particular, people have wanted to know which is the most advantageous gamble in which to partake. The theory of Monetary expectations was developed to a large degree to satisfy this requirement. It represents the earliest theory as recognized in the literature of risk analysis. This concept was prevalent until the mid seventeen hundreds.
The theory of monetary expectation takes the probabilities of all of the events that are possible with a particular gamble and multiplies them by their respective outcomes (in dollars). If the individual is behaving in a rational manner, the choice with the highest monetary expectation is the one chosen.

This theory represents a very objective way of perceiving how people react to risk. It assumes that two consumers encountering an identical gamble would both evaluate and choose the outcome, with no differences in their perceptions of the anticipated risks or the final outcome [Bernoulli 1756]. This concept of taking the highest mathematical expectation was justified by the law of large numbers i.e., the continual participation in a gamble which favors the participant with odds greater than 50 percent will eventually result in the individual having a greater number of wins than losses [Halvorson 1981].

Savage [1954] has listed the major drawbacks of the theory. The first problem concerns the implication that everyone is affected by risk to the same magnitude, regardless of the size of the bet. The consequence of losing a fifty dollar bet for most college students is, in general, greater than the consequence associated with the same bet for a Senator. A second problem with mathematical expectations surrounds its limitations concerning universal applicability. People indulging in making 'poor' gambles and people carrying insurance are a few of the cases that have been cited to indicate that the theory of mathematical expectations has only a limited applicability.
Bernoulli's Analysis

If people did not always choose the decision which would bring to them the highest expected monetary value, in what manner did they act? Daniel Bernoulli [1756] used the terms risk aversion and risk seeking to explain this alternative behavior. Risk aversion occurs when a person has a preference for a sure gain over a gamble that has higher or equal monetary expectation. Risk seeking is when an act of certainty is rejected in favor of a gamble with lower or equal monetary expectation. An example of risk seeking would be when an individual, faced with a decision involving two options, one being a gift of 10 dollars and the other a gamble which has a monetary expectation of 10 dollars or less, chooses the gamble. An example involving risk aversion is when an individual chooses the gift of 10 dollars over a gamble which involves a monetary expectation greater than 10 dollars. Whenever a person makes a risk averse or risk seeking choice, the highest monetary expectation is foregone.

Bernoulli put forth the idea that people maximized utility. He stated: "No valid measurement of the value of a risk can be obtained without consideration given to its utility" (the utility of a possible gain or loss) [Bernoulli 1756 p.23]. He took the concept of utility and substituted it for the dollar outcome when deriving expected values. Bernoulli believed that the same rule or thought cannot be used to evaluate all men's reactions to risk. This was because people have different values or utilities associated with monetary outcomes. These utilities are a function of their total income.
According to expected utility theory, each outcome has some intrinsic value which varies depending on the individual. The utility of each outcome is multiplied or weighted by its probability of occurrence. If the consumer is behaving in a rational manner, he chooses the outcome which has the highest expected utility.

Bernoulli's analysis changed the study of risk. It depicts man as reacting in a subjective rather than an objective manner when confronted with a decision. He thought of utility in cardinal terms, relating it to states of wealth. His thesis was that wealth and utility are positively related. Thus, the larger winnings from a gamble correspond to a larger level of utility. However, this utility would behave according to the law of diminishing marginal utility. This leads to an important concept of Bernoulli's theory: "gains and or losses are a function of the state of wealth ... [in which] ... the person currently exists" [Bernoulli 1956 p.27]. This assumption is a major difference between expected utility theory and the theory that is presented later in the paper. Bernoulli felt that the above assumptions explained why people behaved in either a risk averse or risk seeking manner.

Consider the following example which further illustrates Bernoulli's ideas. A very poor person and a wealthy person are both confronted with the same choices. They have the option of acquiring either a ticket which enables them to a fifty percent chance of winning one hundred dollars and a fifty percent chance of winning nothing, or thirty dollars in cash. In most cases the thirty dollars is very significant to the poor man. Because his total wealth is small, the
addition of thirty dollars is very significant. It has a very high utility for him. More often than not the poor man will choose the risk averse choice. He evaluates the utility of a sure thirty dollars higher than the utility of one hundred dollars multiplied by one half. The wealthy man will, in most cases, hold onto the ticket. A thirty dollar gain is relatively insignificant to a person with a large state of wealth. This example indicates: 1) The decision with the highest monetary expectation is not necessarily the decision with the highest expected utility; 2) Individuals derive their expected utility from their initial wealth state.

Bernoulli's analysis can also be illustrated graphically. See figure 2-4. Let the horizontal axis AR show gains in wealth and the vertical axis AQ show utility. The function BS has a negative second derivative over this range. Let the initial wealth position be AC and a gain to be CD which is a very small amount. The gain in utility which is attributable to the CD change in wealth is RH. But, it is clearly discernable that the amount of utility which is realized from CD is also related to its initial wealth position AC.

Bernoulli has contributed a large amount of work to risk analysis. His theorem was dogma for over two hundred years. It was not until the late 1940's and early 1950's that dissatisfaction appeared. The two major criticisms of Bernoulli's analysis are summarized below.
Writing in 1948, Friedman and Savage showed that Bernoulli's theorem does not account for people making a fair gamble. The problem is due to diminishing marginal utility. Quoting from the authors: "The rejection of utility maximization for decisions under risk is because of the belief in diminishing marginal utility. Diminishing marginal utility plus utility maximization implies that individuals would always have to be paid to induce them to bear risk" [Friedman and Savage 1948 p.280]. For example, using Bernoulli's analysis, if a person is confronted with a fair bet for a dollar, the person would never accept the gamble because the additional dollar he could win would be worth less to him than the dollar he could lose.

There is a second problem with Bernoulli's work caused by his use of diminishing marginal utility. This concept created an illusion of having some objectively measurable quality in goods. The rise of the indifference-curve view of utility resulted in utility ceasing to have any objective significance [Arrow 1951 p.423].

In a later book, Savage [1954] provided a further critique. Bernoulli put forth no reason for supposing that preferences correspond to the expected value of some function, with the result being that more general possibilities must be considered. Savage also felt that features besides expected value functions, such as the range, skewness, etc., should be used in determining preferences.
FIGURE 2-4
Expected Utility Analysis

utility per unit of time

wealth per unit of time
Von Neumann and Morgenstern's Analysis

In 1944, von Neumann and Morgenstern finished a book which challenged the rejection of expected utility theory. They believe that the consumer's objective is to maximize expected utility. The authors developed postulates that, if met, lead to the maximization of expected utility. These postulates are discussed in von Neumann and Morgenstern [1944 pp.40-42] and in Friedman and Savage [1948 pp.279-304]. Arrow states about these postulates: "von Neumann and Morgenstern enunciated a clear set of axioms on choice among probability distributions which led to the assumption of maximizing expected utility and which were convincing" [Arrow 1951 p.424].

Briefly, these are the postulates which von Neumann and Morgenstern formulated. These postulates enabled them to build a conceptual structure for the treatment of numerical utilities. Let 'u', 'v', and 'w' represent distinct and different commodity bundles. The first axiom is: a complete ordering of utilities. This means that for two utilities, there are three possible relationships. The commodity bundle 'u' may be preferred to or indifferent to the bundle 'v', or the commodity bundle 'v' may be preferred to the commodity bundle 'u'. However, only one of these relationships may exist at any given time period.

The second axiom is concerned with ordering and combining. For example, if 'u' is preferred to 'v' and 'v' is preferred to 'w', then without any more information we know that 'u' must be preferred to 'w'.

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This is also known as the axiom of transitivity.

The third axiom is contained under the heading of algebra of combining. It stipulates that it is irrelevant which order the preferences of choice are presented to a decision maker. This is also referred to as the postulate of invariance. The role of invariance in the decision making process plays a major part in the latter segments of this paper.

There are a number of assumptions which have been added to the von Neumann and Morgenstern postulates. These are summarized below. In defining the domain of the expected utility function (U), most economists include the risk aversion assumption; \( U'' < 0 \). They also assume that the overall utility of a gamble is the sum of the expected utility of its outcomes, and the utility resulting from integrating a gamble with one's assets exceeds the utility of those assets alone. This results in the domain of the utility function being the total assets of the individual [Kahneman and Tversky 1979 p.264].

Von Neumann and Morgenstern state that the expected utility rule is the mathematical expectation or probability - weighted average of the utilities of the associated consequences or outcomes. The amount of uncertainty of a particular decision is reflected by the dispersion of the probability weights over the possible outcomes [Hirshleifer and Riley 1979]. A main advantage of von Neumann and Morgenstern's theory is that no matter how complicated the structure of any particular decision, it is always possible to describe the expected utility
function by a single probability distribution of the final outcomes, assuming that all of the aspects of the postulates are satisfied. Given their assumptions, utility becomes a number up to a linear transformation. "Their theorem gives a method of assigning utilities to individual outcomes so that the utility assigned to any probability distribution is the expected value under that distribution of the utility of the outcome" [Arrow 1951 p.425].

Other Analyses

In most aspects, the von Neumann and Morgenstern theory is the most commonly used theory of risk at present. However, there have been other theorems. Friedman and Savage [1948] have presented a a theory in which people react in both a risk averse and a risk seeking manner, depending on the size of the gamble or risk and the current state of their total wealth. Halvorson and Ruby [1981] feel that expected utility theory and expected monetary value are both valid under certain conditions. They have designed rules which determine the magnitude of the errors involved in using both of the theories with the result being that they have guidelines for when one theory is preferred over the other.

Comparison of Analyses

The theories which were presented above differ from one another in many aspects. However, they do have a few points in common. The first point is that the formulation of various commodity choices, under uncertainty, are based on specified "states" [Hirshleifer and Riley 1979 p.1376]. In other words, an individual bases his choice on what state
A second point is that in general, economists using any of the above analysis assume that most individuals behave in a risk averse manner. Halvorson states: "The principle empirical evidence in favor of people being risk averse is that people; prefer to hold a variety of assets". [Halvorson and Ruby 1981 p.68]. That is, people prefer a portfolio which encompasses many assets to one single asset which may have a higher earning ratio. Although noting some glaring discrepancies, Hirshleifer and Ruby assume that risk aversion is the normal mode for individuals [Hirshleifer and Ruby 1979 p.1382].

What is the reason for bringing forth these two points? The analysis which is presented in the following section is based upon an assumption that differs from point one above. This difference results in the conclusion that people do not usually behave in a risk averse manner. In turn, this result may then shed some light on the reason why equivalent and compensating variation differ empirically despite what conventional theory suggests.

2.5 Prospect Theory

Through the years, a number of empirical discrepancies have been observed regarding deviations from what expected utility theory would predict as correct behaviour. Many economists have noted problems with expected utility theory [Slovic and Lichenstein 1983 p.597]. One of the more troublesome discrepancies pertaining to decision theory deals with
preference reversals. To understand preference reversals, consider a person who is confronted with two gambles. The first gamble offers a high probability of winning a small amount of money. The second gamble consists of a low probability of winning a large amount of money. Even though gamble number two is assumed to have a higher monetary expectation, most people choose the first gamble. It seems that a near certain outcome outweighs a larger expected monetary reward. This decision contradicts ... "almost all theories of preference, including the expected utility theorem" [Slovic and Lichenstein 1983 p.596].

Most economists who have worked on this problem feel that the reason people are making choices which are not compatible with expected utility theory is that the experiments themselves are poorly designed [Reilly 1982 and Grether and Platt 1979]. However, Slovic and Lichenstein believe: "Reversals are not an isolated phenomenon but one of a large class of findings that demonstrate violations of preference models" [Slovic and Lichenstein 1983 p.567]. They feel that it would be worthwhile to look at new proposals or theories as well as fine tuning experiments to test the existing theory.

Kahneman and Tversky have described several classes of problems concerning decisions, both hypothetical and real i.e., involving sums of cash, in which preference decisions systematically violate the axioms of expected utility theory [Kahneman and Tversky 1979, 1982, and 1984]. The lack of explanatory power for real events led Kahneman and Tversky to propose an alternative idea called prospect theory. Many of prospect theories assumptions contradict the assumptions of expected utility.
Prospect Theory: How Decisions are Analyzed

When people are faced with a decision involving uncertain outcomes, how do they decide? It has been assumed in theories of rational decision making that people take a comprehensive look at the outcomes and are cognizant of the effects on their total welfare. For example, it is assumed that people perceive an even bet for ten dollars as a choice between no change in total wealth or an even chance to increase or decrease total wealth by ten dollars [Kahneman and Tversky 1984].

According to prospect theory, people adopt a limited view of the outcomes of decisions; they identify potential outcomes merely as gains or losses relative to a neutral point. The limited view point which is assumed in this theory recognizes the fact that people in general do not have the time or information to make comprehensive decisions.

Concerning prospect theory, two phases of cognitive activity are distinguished when a choice is contemplated: an editing and an evaluation phase. The editing phase consists of simplifying or organizing and reformulating the available options. This stage transforms the outcomes and probabilities associated with the decision into terms of gains, losses, and the maintenance of the status quo, instead of the presently accepted theory of changes in states of wealth.

Some editing operations appear to prevent or inhibit the organizing and reformulating of other options. This can result in inconsistent decisions, based upon what part of the decision is edited first, which in turn is a function of how the initial problem is framed [Kahneman and
Tversky 1979 p.274]. Framing deals with the way a decision is formulated in words. Decisions may be phrased so that a person perceives his choice to be either a gain or a loss.

Decision Weights

The evaluation phase consists of analyzing the edited information and choosing the outcome with the highest utility. The outcomes are measured values of the gains and losses. Each edited outcome has a probability of occurrence. Attached to these probabilities are decision weights. These decision weights are assumed to be nonlinear and reflect the impact of each probability on the overall value of the prospect. The slope of a decision weight function can be perceived as a measure of the sensitivity of preferences to changes in probability.

These nonlinear decision weights reflect the phenomenon of individuals putting different emphases on probabilities. This reflects the common perception that most people would pay more to improve their chances of winning from 90 percent to certainty than they would to raise their chances from 40 to 50 percent. It is also argued that an increase from impossibility to 10 percent is more significant than an increase from 30 to 40 percent [Kahneman and Tversky 1982 p.164].

Kahneman and Tversky theorize that moderate and high probabilities are underweighted relative to sure things. This contributes to risk aversion in gains by reducing the attractiveness of gambles and contributes to risk seeking in losses by diminishing the aversiveness of negative gambles. Conversely, low probabilities are overweighted
resulting in an enhancement of small probability wagers and amplifies the aversiveness of a small chance of a severe loss [Kahneman and Tversky 1984 p.345]. Implicit to these assumptions is the hypothesis that outcomes which are obtained with certainty are over weighted relative to uncertain outcomes. The idea that people evaluate various levels of probability differently and in the above manner seems to explain the problem of preference reversals.

Value Curve

The assumption that people generally react toward a decision in a risk averse manner has been widely held by economists. In contrast, psychological studies have shown that risk seeking preferences are common when people must choose between a sure loss and a substantial probability of a larger loss [Kahneman and Tversky 1982 p.160]. These studies, which have encompassed both hypothetical and real situations in many different areas, led the authors to propose that preferences between gains are risk averse and preferences between losses are risk seeking. This gives rise to a decision making function which differs from the function which is associated with expected utility theory. For an example of this "new" type of decision making function see figure 2-5. These functions relate subjective values (on the vertical axis) to objective outcomes such as money (on the horizontal axis). In prospect theory, a concave, convex downward decision making function is present as opposed to a concave function which occurs in expected utility theory.
The concave section of the function is for gains, showing that for each extra dollar gained there is less value accrued than there was from the previous dollar. This conveys the impression that, "the difference between a gain of $100 and one of $200 is more significant than the difference between a gain of $1100 and one $1200" [Kahneman and Tversky 1982 p.162]. Concavity favors risk aversion.

Risk seeking is represented in the convex portion of the value function and under the domain of perceived losses. Convexity illustrates the assumption that each dollar lost causes a smaller change in value than the preceding one, up to an amount that would be viewed as a catastrophic loss. This is in accord with the common perception that the difference between a loss of $100 and one of $200 is more significant than the difference between a loss of $1100 and $1200.

An important part of prospect theory as it pertains to this paper is the shape of the decision making function. A common sense assumption with great import is that a loss of X dollars is viewed much more adversely than an equivalent gain is viewed positively. This is called loss aversion. For example, Kahneman and Tversky conducted a study which showed that participation in a fair bet in which an individual has the possibility of losing $10 would be unacceptable unless there was the possibility to win at least $30 [Kahneman and Tversky 1982 p.342]. The asymmetry of the decision function in relating objectively equivalent gains and losses is illustrated in the greater steepness of the value function for losses. In figure 2-5 the distances AN and NB denote an equivalent (in monetary terms) loss and gain, respectively. However,
because of the shape of the value curve, note the disparity between the two measures of utility, NC and ND.

In summary, the value function has three attributes that are ascribed to it, each with important ramifications. They are: 1) the function is based upon analyzing how people react to perceived gains or losses, relative to a neutral point, not to total wealth; 2) people are usually risk averse for gains and risk seeking for losses which results in a function that is concave for gains and convex for losses; 3) the convex portion of the function is considerably steeper than the concave portion, reflecting loss aversion, which means that people view a loss more significantly than they do an equivalent gain.

An example of the effects of framing and the greater steepness for losses on the decision function can be seen by observing gas stations and their "discounts" for cash payments. Thaler in Kahneman and Tversky noted: "lobbyists for the credit card industry insisted that any price difference between cash and credit purchases be labeled cash discount rather than a credit card surcharge. The two labels frame the price difference as a gain or as a loss by implicitly designating either the lower or the higher price as normal. Because losses loom larger than gains, consumers were less likely to accept a surcharge than to forego a discount" [Kahneman and Tversky 1984 p.346].
FIGURE 2-5
Value Curve
A few examples will help to clarify the major concepts of prospect theory. Assume that an individual is confronted with two choices. This person must choose between a 99 percent chance of winning $10.00 and a 60 percent chance of winning $20.00. The latter choice has the highest expected monetary value and the individual is expected to choose this decision. However, when decision weights are considered, it is possible that an individual might choose the former choice. This would be the case if the individual's decision weight for the near certain outcome was sufficiently large enough in comparison to the decision weight placed upon the 60 percent outcome. For example, if the individuals' decision weight for the 99 percent chance of winning $10.00 is one and the decision weight for the 60 percent chance of winning $20.00 is .7, the former choice would be chosen.

The following example clarifies the impact which framing bias and the value curve have on decision making and the two measures of consumer's surplus. There are at least two ways of framing a question: from a gain or a loss perspective. In this example, assume that the individual is not satiated with the good. It is assumed in prospect theory that if an individual is confronted with the decision: What is the maximum amount of money would you be willing to pay to be able to consume good X, given that X's price has just been decreased relative to all other prices? This question is framed as a gain; the individual has the opportunity of consuming the same amount or more of X at the lower price. Alternatively, if the individual is confronted with the decision: What is the minimum amount of money that you would accept to
forego the opportunity of consuming good X at the lower price level, they would be faced with a question framed as a loss. In accord with the value curve and the loss aversion principle, the latter question will elicit a higher dollar amount. This is because of the hypothesis that individuals require more compensation when confronted with a potential loss than they do when they are confronted with an objectively equivalent potential gain. Readers should note that the above two questions mirror the definitions of compensating and equivalent variation that were given earlier in this chapter. Thus, prospect theory presents a viable hypothesis as to why compensating and equivalent variation differ when empirically measured. This concept will be explored further in the following section.

2.6 Prospect Theory and Consumer's Surplus

The value function and all of its associated implications have not been put forth as a universal law of individuals' behaviour. It is presented as a summary of aggregate behaviour under uncertainty. The implications of prospect theory on the empirical discrepancy between equivalent and compensating variation are shown. It is assumed that prospect theory is valid in the aggregate. However, this does not imply: 1) The convexity changes of an individuals' value curve are uniform across all individuals; or 2) The neutral point from which decisions are made is uniform across individuals.
To reiterate the problem which this paper is concerned: a person who uses a particular fishing site is asked two questions. First, what is the maximum amount they would be willing to pay to continue to use this spot and second, what is the minimum amount they would accept as payment to forego the use of the site. According to economic theory, the two measures of value, compensating and equivalent variation respectively, should be nearly equal with any small discrepancy due to income effects. However, empirical analysis has shown that there is a large difference between the two measures. In many studies, equivalent variation has differed from compensating variation by four to twenty times.

If we look at the above questions objectively or in terms of total wealth changes, the two questions should elicit equal answers. Assume that the fishing site has a fixed market value and each outcome has an equal opportunity of occurring. In both cases it is a simple trade off between goods. For example, if you would trade one orange for two apples, you would also be willing to trade two apples for one orange and suffer no loss of welfare, at least for one exchange.

It is proposed that prospect theory offers an explanation as to why the two measures of value differ. With prospect theory, we would analyze the problem differently. When confronted with the two questions, we would view them as a possible gain or a possible loss of land respectively. Even though the market has fixed the price of the fishing site i.e., the gain and the loss of the site in monetary terms are equal, prospect theory states that people view a loss more adversely.
than they do an equivalent gain. The phenomenon of loss aversion results in a higher negative value of the loss of the site than the positive value associated with the use of the site.

If people view the loss more adversely, it makes sense that they demand a higher level of compensation to make them feel as well off. The degree of compensation is a function of the steepness of the risk seeking portion of the curve relative to the slope of the risk averse part of the curve. The more a person behaves in a loss averse manner the steeper the risk seeking portion of the curve becomes. Therefore, it would seem the greater the difference in slopes between the risk averse and risk seeking segments of the curve, the greater the difference between compensating and equivalent variation.

The question of why people view a loss so much stronger than an equivalent gain is just beginning to be explored. Knetsch is working on a perspective that people over emphasize the possession of property rights. Thaler has coined a term, the endowment effect, to acknowledge this behaviour [Kahneman and Tversky 1984 p.348].

While prospect theory is in its infancy, it is believed that using this theory to interpret individuals' behaviour explains at least some of the variation in the two measures of value. This type of analysis has been shown to correctly predict behaviour in some recent studies. In the domain of economics, Fishburn and Kochenberger [1979] analyzed thirty empirically assessed utility functions which were derived from five different sources. These functions dealt with changes in wealth.
After finding mathematical functions to fit the data, they found that most utility functions for gains were concave, most utility functions for losses were convex, and only three cases showed risk aversion for both gains and losses. Most important for my analysis, with only a single exception, utility functions were considerably steeper for losses than for gains.

As far as I have been able to ascertain, there has been no study to date which has attempted to test the effectiveness of prospect theory in allaying the problems associated with the measurement of compensating and equivalent variation. In the following chapter an empirical study is outlined. This study is undertaken in an attempt to test the effectiveness of prospect theory in explaining some of the disparity which is associated with the two measurements of consumers surplus.
CHAPTER III

BACKGROUND FOR THE EMPIRICAL ANALYSIS

3.1 Introduction

It is proposed that compensating variation differs from equivalent variation because of the failure of a hypothesis within decision theory. This hypothesis is known as invariance. Kahneman and Tversky [1984] first mentioned this in a paper which considered the way individuals make decisions in risky and riskless contexts.

Invariance can be perceived as a rule which requires that a person's preference ordering of outcomes not be a function of the way in which they are described. In other words, two options which are equivalent when shown together should elicit the same preference when shown separately [Kahneman and Tversky 1984 p.343]. For example, when patients are confronted with a choice on whether to have an operation or not, their decision depends upon how the question is poised. If they are told they have a 70 percent chance of living through the operation they are more inclined to proceed with the operation than if they are told that they have a 30 percent chance of dying [Kahneman and Tversky 1984].
In prospect theory, it is proposed that individuals make decisions based on how the problem is presented to them; i.e., they make a decision based on whether they perceive the decision to involve a gain or a loss. If this hypothesis is correct, it means that the hypothesis of invariance is not compatible with the way individuals make decisions. Investigating the validity of the invariance hypothesis is the focus of the empirical portion of this paper.

Kahneman and Tversky state that the failure of invariance has two causes: framing bias and decision weights. In numerous empirical experiments, both framing bias and decision weights have been shown to invalidate the hypothesis of invariance [Kahneman and Tversky 1984]. These experiments presented individuals with choices involving varying levels of probability of occurrence and with varying outcomes.

This paper is going to test the validity of these hypotheses within the framework of an economic survey involving compensating and equivalent variation. This economic study entails interviewing a set number of respondents with a questionnaire designed to elicit information regarding framing bias and decision weights. Before delving into this empirical model, it is necessary to explain the two causes.
3.2 The Hypotheses

Framing Bias

Framing bias refers to the phenomenon of phrasing objectively equivalent questions in such a manner that the answers which are elicited differ. By objectively equivalent I will be referring to expected monetary outcomes. This does not have to be the case, however. For example, a decision is made to use an antidote to cure an outbreak of a fatal disease. It will result in a recovery rate of 40 percent. This statement may also be phrased in the following manner: the use of the antidote will result in a 60 percent death rate. These two statements are also viewed as being objectively equivalent.

To have framing bias, there must be a reference point from which the decision is being analyzed. For example, a decision can be framed as either a gain, loss, or a maintenance of the status quo, with differing answers expected depending upon which reference point is used. This difference is due to the belief that an individual places a higher value on a gamble involving a perceived loss than they do on a perceived gamble involving an equivalent gain.

An outcome may also be framed so that asset positions which incorporate initial wealth into the decision making process are used as the reference point for deciding the outcome. The decision is based on whether the outcome is a gain or loss in accord with the assets which the individual possesses. This is the current position which economic
theory takes [Kahneman and Tversky 1984].

In empirical tests, Kahneman and Tversky have found the phenomenon of framing bias to be both: "pervasive and robust, as common among sophisticated respondents as among naive ones" [Kahneman and Tversky 1984 p.343].

Decision Weights

Most studies which combine probability and cardinal utility analysis use, in some manner, expected utility analysis. This method dates back to Bernoulli. Studies which use von Neumann and Morgenstern's analysis incorporate expected utility into their analysis. See for example Green's [1961] and Swalm's [1966] studies where they derived utility functions of businessmen. Von Neumann and Morgenstern's analysis concerning expected utility theory gives unique utility measures up to an arbitrary linear transformation.

Kahneman and Tversky put forth the hypothesis that this value is not a linear function of the probability of an outcome [Kahneman and Tversky 1984]. Instead, they came up with a weighting function which is dependent upon probability but not in a linear fashion. For example, they surmise that an increase in the probability of an event occurring from zero percent to five percent has a greater impact upon the value of an outcome than does an increase from 35 percent to 40 percent. They also believe that an increase from 95 percent to 100 percent has a greater impact than does the zero percent to five percent.
Kahneman and Tversky derived a hypothetical curve for an individual which plots the decision weight attached to an outcome as a function of its probability. See figure 3-1. This curve, except for the endpoints, illustrates that an increase in the probability of the outcome occurring raises the value of the outcome by less than the respective increase.

3.3 Empirical Framework

Background

In Missoula, Montana, until mid 1983, Rattlesnake Creek and wells were the only sources of water. Depending upon the location within the city, individuals could consume either of the two possibilities. It was also possible that during certain months of the year Rattlesnake water consumers would be switched to well water. This would occur during times of high turbidity, extremely cold temperatures, or very low water levels. During the summer of 1983, Giardia was found in the Rattlesnake water supply. This resulted in the entire city being hooked up to well water.
FIGURE 3-1
Hypothetical Weighting Function
Since well water has become the sole source of water, there have been complaints about the hardness of the water. Missoula's well water has been tested and found to be seven times as hard as Rattlesnake water. Hardness can be measured by the amount of minerals (measured in parts per million), in the water. The most common minerals found in Missoula's water supply are calcium carbonate and magnesium. The harder water results in deposits building up in items such as faucets, pipes, and dish washers. For some home owners this problem has necessitated the purchase of water softeners. In conversations with store managers which sell water softeners, they mentioned a definite rise in the sales of this product since the summer of 1983.

Several studies have been completed which detail the economic costs of a new water treatment plant. This would enable Rattlesnake water to be consumed once again. One study, funded by the Mountain Water Company, found that the annuity required to fund the treatment plant plus operating and maintenance costs for a 50 year time period was approximately $767,230.00 per year [Park Water Co. Memorandum 1984].

Objectives of Analysis

The empirical analysis focuses on three objectives. The first objective is to derive an empirical estimate of the average dollar value placed upon Rattlesnake drinking water by Missoula's water consumers. Both compensating and equivalent variation dollar values will be measured. The second objective is to check for framing bias. If it is found to be present, its effect upon the two measures of value will be
The last objective is to build a decision weight function and find out how the probabilities are weighted.

Dollar estimates of compensating and equivalent variation will be derived for three reasons. Their values will indicate what the preference is toward consuming Rattlesnake water which in turn reflects the respondents' attitudes toward building a water treatment plant. However, it should be noted that this is not a measure of the total benefits which would be derived from the installation of a treatment plant. The second reason is to find out what variables influence the two measures of value. The third reason is to find out what type of relationship exists between the two measures of value. For example, this study will determine if one value is greater than or equal to the other. In other words, do these values correspond to each other in the way economic theory dictates or do they follow the relationship which has developed in similar economic studies.

The second objective is concerned with framing bias. The relevant questions are formulated so that it will be possible to detect if the hypothesis of invariance is broken. This is accomplished by deliberately framing a decision problem in more than one way. If framing bias is found to be present, it will give credence to Kahneman and Tversky's assertion that outcomes may be manipulated, "without distorting or suppressing information, merely by the framing of outcomes and contingencies" [Kahneman and Tversky 1984 p.346].
In this study compensating variation is the variable manipulated. It is framed from both a gain and a loss perspective. The questions which elicit the compensating variation and frame responses are shown below.

4) Assume that it would be possible to consume Rattlesnake water if a water treatment plant is built. What is the maximum amount which you would be willing to have added on to your existing monthly water bill to give you the opportunity to be able to consume Rattlesnake rather than well water?

6) Imagine that you are currently consuming Rattlesnake water and the water company issues a report which states that they will be forced to shut off Rattlesnake water next month due to high costs unless consumers would be willing to pay higher water bills. What is the maximum increase in your monthly water bill that you would allow in order that you continue the Rattlesnake water service?

Both questions refer to a hypothetical situation. In question four the consumer is asked to pay when the reference point is perceived to be a gain. Question six is framed so that the consumer is asked to pay when the reference point is perceived to be a prevention of a loss.

The hypothesis that risk plays a role in the disparity between compensating and equivalent variation will be tested by the decision weight function. The goal of this objective is to find out how the value of drinking Rattlesnake water changes as the probability or risk of depleting Missoula's aquifer is varied. Two functions will be derived, one for each measure of value. Of major interest will be determining if there is nonlinearity within either of the decision weight functions. Also, the two functions will be compared to see how
they relate to one another. This decision weight function may also indicate that the hypothesis of invariance does not work under normal conditions of human decision making.

Questionnaire

The questionnaire was designed to derive the data necessary to satisfy the three objectives. Appendix A contains more detailed information about the questionnaire. Questions numbered one, two, three, four, seven, eleven, and twelve all identify potential independent variables that could have an impact upon either an equivalent or compensating valuation value. Using both equivalent and compensating variation as the dependent variables, an explanatory equation will be derived with the above questions as the independent variables. This equation will be derived using regression analysis.

Question number four measures compensating variation and number eight measures equivalent variation. These questions do not have the variable risk added explicitly. Questions nine and ten ask equivalent and compensated variation values, respectively. These two questions do have various risk components explicitly included.
Sampling Procedures

Three problems surfaced when the questionnaire was distributed. The first problem dealt with who was going to receive the questionnaire while the second problem involved finding a proper sample size. The third problem was how to obtain a sample that was random.

It was decided to constrain the population to the boundaries of Missoula with three exceptions. The upper portions of the Rattlesnake which are not within the city limits but which are serviced by Mountain Water Company were included in the population. The area of the South Hills which is bounded by 39th Street, Highland Park, and 25th Avenue was excluded from the sample. This was because many of the residences have their own water supplies. It is believed that the best sample would include only those who purchased water from Mountain Water Company. The third problem dealt with the area West of Reserve Street. Portions of this area are within the city limits and are serviced by Mountain Water Company but they were excluded because of problems of attaining a random sample. This problem concerned the farms which encompassed very large areas. This would have resulted in this area having a disproportionate effect upon the sample. In order to maintain the assumption of land area being a proxy for population percentages, the decision was made to exclude this area. Being constrained by limited resources such as money and time, it was felt that the assumption of transposing the percentage of geographical area into an equal percentage of consumers is realistic.

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Once the area of the sample was defined, it had to be determined who within a chosen residence was eligible to answer the survey. It was decided that an individual would be acceptable only if he or she was intimately involved with the payment of bills for the household. It was felt that a person who was familiar with the dollar amounts of utility bills, in particular the water bill, was more desirable. This was important because an iterative bidding device for the equivalent and compensating variation was not used. The individuals' water bill was intended to give them some sort of framework from which to give a bid. The second reason was to acquire a more sophisticated response in all aspects of the questionnaire.

In choosing a sample size, the main criteria was that the sample was large enough to be able to apply statistical analysis. This study was not concerned with making the analysis policy defensible. This would require a sample size larger than the one which was chosen. Statistical theory states that if the sample size is greater than or equal to 30 the sample estimate of the variance is a good approximation to the unknown population variance [Koutsoyiannis 1977]. It was decided to give 35 interviews, giving an error margin of five.

In delivering the questionnaire, the second problem that was encountered concerned the derivation of a sample that was random. Geographically, approximately 60 percent of Missoula was on the Rattlesnake water system. The assumption was made that this geographical percentage applies equally to the number of water consumers. Thus, for this sample 35 observations were chosen from a
pooled weighted average which contained the following ratio: 40 percent of households which live in areas that were unable to consume Rattlesnake water and 60 percent of households which lived in areas which were able to consume Rattlesnake water.

In the actual derivation of this sample, there were 13 residences that were able to consume only well water and 22 residences that were able to consume Rattlesnake water. These numbers were compiled from an appropriately weighted random numbers table.

After compiling the appropriate ratio of residences which were bounded by their respective water capabilities, 13 blocks were chosen from within the well water region and 22 blocks were chosen from within the Rattlesnake region. The following procedure was used to determine which residence to interview on the randomly picked blocks. Starting on the North West corner of the chosen block and proceeding in a clock wise direction, each residence was counted and recorded numerically on a slip of paper. After counting the number of residences on the block the numbers were placed in a box and one was picked at random.

If the individual at the chosen residence refused to be interviewed, the random draw was repeated for the same block. If no one was home at the chosen residence, the address was recorded and an attempt was made at a later date. If this later attempt also failed, another residence was chosen from the same block in the above manner.
Another problem which occurred was when the chosen site turned out to be a commercial venture. When this happened, a new site was picked or in the case of the entire block being commercial, a new random block was chosen from the same region. Residences were also encountered which contained individuals who were recent arrivals in Missoula. It was decided to interview them anyway on the grounds that they would still be able to express a preference.

It is possible that the small sample size of this study can lead to a distorted statistical analysis. However, the objective of this study is to design a small scale experiment which tests a particular solution to an important economic anomaly. If the empirical analysis from this study gives credence to the hypotheses which are being analyzed, a more in depth empirical analysis should be carried out.

It is felt that any error which results from the above assumptions will be very small, especially in respect to the direction which this study takes. If this study is random, it will also be free of bias, which is the position taken with the statistical analysis.
CHAPTER IV

STATISTICAL ANALYSES

4.1 Introduction

This chapter contains the statistical analysis for this paper. There are four sections, each one dealing with a specific area. The second section analyzes compensating and equivalent variation plus framing bias. These variables do not have risk included. Their values are tested to make sure they are significantly different from zero. This section also deals with compensating and equivalent variation values which have varying levels of risk explicitly added. The mean values are calculated for all of the risk levels. The means are then compared and tested for significance.

In sections three and four, the variation in compensating and equivalent variation values are observed as an added variable - risk - is added to the analysis. The third section is concerned with the calculation of these risk varied values. A decision weight function is derived in the fourth section. This section builds a function and calculates the associated probability weights for each value of risk. It will show if the various risk levels have weights that result in either under or overweighting outcomes. Section five is concerned with the analysis of the interrelationships between the independent and the dependent variables. The technique of regression analysis was
originally going to be used. However, due to the large amount of heteroscedasticity which was found to exist within the explanatory variables, the technique of factor analysis was finally employed.

4.2: Significance of the Variables WTP, WTS, and FRAME

The variables WTP, WTS, and FRAME measure compensating variation, equivalent variation and framing bias, respectively. The variable WTP measures the maximum dollar figure which the people from this sample would be willing to pay in the form of increased water bills in order to obtain the use of Rattlesnake drinking water. The average or mean dollar figure is $2.80 with a standard error of 0.611. Using the statistic student's T, this mean figure is statistically significant or different from zero at the 0.999 confidence level. This was calculated using a two tailed test with 32 observations.

The variable WTS measures the minimum dollar amount which consumers from the sample felt was necessary to compensate them for not being able to consume Rattlesnake water. This amount was to be deducted from their water bill. The mean value is $6.40 with a standard error of 1.71. The WTS mean is statistically different from zero at the 0.999 confidence level and was calculated using a two tailed test with 32 observations.
The variable FRAME was inserted into the questionnaire to ascertain whether or not two questions (which in objective terms requested equivalent payments) would elicit different amounts based upon the way the question was framed. The mean bid was $4.09 with a standard error of 0.854. Calculated using a two tailed test with 32 observations, this mean is also statistically different from zero at the 0.999 confidence interval.

Relationships Between WTP, WTS, and FRAME

I was interested in the relationships between the three variables. Specifically, I wanted to determine if any statistical difference between the three means existed or whether the variations within the means were due to the randomness of the experiment. The null hypothesis in all of the statistical tests below is: no difference between the means or the difference between the means is equal to zero. A correct statistical procedure to test this hypothesis is paired samples which uses the student T [Wonnacott and Wonnacott 1977 p.216]. Paired samples is a test that is appropriate for variables which are not independent of one another. The results discussed below are summarized in table 4-1.

In comparing WTS and WTP, economic theory states that the null hypothesis should be accepted. My results turned out contrary to economic theory. At the 97.6 percent level of confidence, the null hypothesis of no difference between the means is rejected. The confidence interval was between 0.0037 and 7.199. The possibility that the two means are equal seems implausible because the null hypothesis
falls outside this interval.

In objective terms, the variables WTP and FRAME should measure the same dollar amount. As a result, it is expected that the null hypothesis should be accepted. At the 97.9 percent level of confidence the null hypothesis of no difference between the means is rejected. This confidence interval lies between 0.0018 and 2.567. This result implies that it is highly improbable that the two means are equal which in turn gives credence to the issue of framing bias. It should be noted that in this study the variable which is associated with the framing loss, FRAME, has a larger mean than the one associated with a framing gain, WTP.

Table 4-1: Paired T Tests for the Variables WTP, WTS, and FRAME

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEANS</th>
<th>(DIFFERENCE) MEANS</th>
<th>STANDARD ERROR</th>
<th>T VALUE</th>
<th>2-TAIL PROB.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTS</td>
<td>6.406</td>
<td>3.5966</td>
<td>1.516</td>
<td>2.37</td>
<td>.024</td>
</tr>
<tr>
<td>WTP</td>
<td>2.8041</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRAME</td>
<td>4.084</td>
<td>1.2844</td>
<td>.530</td>
<td>2.42</td>
<td>.021</td>
</tr>
<tr>
<td>WTP</td>
<td>2.8041</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WTS</td>
<td>6.406</td>
<td>2.3122</td>
<td>1.483</td>
<td>1.56</td>
<td>.129</td>
</tr>
<tr>
<td>FRAME</td>
<td>4.084</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

calculated with 32 observations
Since the preceding test has shown an influence that may be attributable to framing bias, an interesting test would be to examine the statistical difference between the variables WTS and FRAME. Both variables are framed as losses. But, in the former payments are received and in the latter payments are made. With this analysis, the null hypothesis of no difference between the two means is accepted at the 88 percent confidence level. It is not until the 87 percent confidence level that the null hypothesis could be rejected. The confidence interval is between 0.00028 and 4.62412.

Outliers

In analyzing the data for the three variables, WTS had one outlier which was for $50.00. This outlier was 4.7 standard deviations from the mean. The next highest value was for $20.00 which was 1.4 standard deviations from the mean. It was decided to retest the relationships between WTP and WTS, and between WTS and FRAME with the seventeenth case from the Rattlesnake region (the $20.00 value) deleted from the data.

The affect on the variable WTS was to lower the standard error by nearly double the percentage that the mean was lowered. The standard error declined by 41 percent while the mean declined by 22 percent. The null hypothesis of no difference between the means of WTS and WTP was rejected at the 99.5 percent confidence level. This resulted in an increase in the confidence interval by 1.9 percent. The null hypothesis that there is no difference between the means of the variables WTS and FRAME was rejected at the 93.2 percent confidence level. This results
4.3 Equivalent and Compensating Variation With Risk

Introduction

This segment of my analysis pertains to deriving empirical estimates of compensating and equivalent variation with the added component of risk. This analysis is divided into two objectives. The first objective is concerned with the comparison of compensating and equivalent variation values to each other when various risk possibilities are introduced. The second objective is to build a decision weight function. It is shown how the value of Rattlesnake water changes as various probabilities of acquifer depletion are introduced. In particular, this study is interested in the type of relationship or function that is developed between varying gradations of risk and their accompanying bids.

Values With Risk Explicit

In the questionnaire, questions numbered nine and ten respectively, derive equivalent and compensating values for Rattlesnake water. These two questions will be referred to as RWTS and RWTP. These questions ask for separate values which correspond to different levels of risk of acquifer depletion. The values of risk vary between 10 and 100 percent with 10 percentage point iterations.
The mean values were calculated for equivalent and compensating variation at the ten different probability levels (see table 4-2). The RWTP mean was calculated using 31 observations and the RWTS mean was calculated using 29 observations. The decrease in observations between these mean bids and the mean bids associated with WTP and WTS was due to nonresponse answers. Individuals found the questions which involved bid iterations much more difficult to answer.

The compensating variation value for a ten percent chance of aquifer depletion is $3.15. Comparing this figure with the compensating variation value of $2.80 which was derived within a riskless context gives credence to both figures. Apparently, a ten percent chance of aquifer depletion does not have a large affect upon the welfare of the respondents. An additional charge of $0.35 per month in their water bills gives them the same level of welfare as the scenario when there was no mention of risk.

However, there is a curious outcome involving the initial equivalent variation values. The risk induced value of 10 percent is less than the value without risk and the 20 percent figure is just equal. This implies that the respondents require less compensation if there is a very small chance of depleting the aquifer as opposed to the question which does not mention the problem of aquifer depletion.

I believe that there may be a plausible explanation for this discrepancy. Respondents may carry some information into the interview process which makes them feel that, in real life, there is a twenty
percent chance of the aquifer being depleted. This phenomenon has been referred to in the literature as information bias. The reason that this does not affect compensating variation could be due to self interest on the respondents behalf.

Comparison of Equivalent and Compensating Means

The equivalent and compensating bids are compared using two different techniques. In one case the means for both categories were computed at each risk level. To determine if the mean values with equal risk of compensating and equivalent variation were different from one another, the paired T sample test was used. In all of these tests the null hypothesis that was tested was; no difference between the two means. The results are in table 4-2. Two points should be observed. All of the paired means are statistically significant or different from zero by at least the 90 percent confidence level. The statistical significance increased after the 30 percent chance of aquifer depletion to the 99 percent level. It stayed at that level throughout the rest of the variations of risk. These mean bids are illustrated in figure 4-1.

The other point that should be noted is that the difference between the respective means grows larger for each successive risk level. This difference is calculated in the following manner. The change from 10 to 20 percent is plotted at the twenty percent level of risk. The change from 20 to 30 percent is plotted at thirty percent. This method is followed throughout.
The difference between mean bids behaves in an oscillating manner up until the eighty percent value for compensating variation and the seventy percent value for equivalent variation. These differences in values may be observed in figure 4-2.

Outliers

In studying the data, case number 22 from the Rattlesnake area has the largest standard deviations from its' means. This is for both compensating and equivalent variation. There are 13 outliers which are greater than or equal to four standard deviations to their respective means. Case number 22 provides 11 of these outliers. This case was removed and the T samples were run again. The two-tailed probabilities do not change for any of the risk levels. However, the mean values are substantially lowered. See figure 4-3.
Table 4-2: Paired T Tests for Variables with Risk

<table>
<thead>
<tr>
<th>%</th>
<th>VARIABLE</th>
<th>MEAN</th>
<th>(DIFFERENCE)</th>
<th>STD</th>
<th>T</th>
<th>2-TAIL</th>
<th>PROB</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Equivalent</td>
<td>4.519</td>
<td>1.369</td>
<td>.788</td>
<td>1.74</td>
<td>.093</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>compensating</td>
<td>3.15</td>
<td>2.427</td>
<td>.887</td>
<td>2.74</td>
<td>.01</td>
<td>*</td>
</tr>
<tr>
<td>20</td>
<td>Equivalent</td>
<td>6.402</td>
<td>3.575</td>
<td>1.352</td>
<td>2.64</td>
<td>.013</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>compensating</td>
<td>3.975</td>
<td>4.288</td>
<td>1.389</td>
<td>3.09</td>
<td>.004</td>
<td>*</td>
</tr>
<tr>
<td>30</td>
<td>Equivalent</td>
<td>8.962</td>
<td>5.349</td>
<td>1.898</td>
<td>3.09</td>
<td>.004</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>compensating</td>
<td>8.19</td>
<td>6.664</td>
<td>1.879</td>
<td>3.55</td>
<td>.001</td>
<td>*</td>
</tr>
<tr>
<td>40</td>
<td>Equivalent</td>
<td>10.712</td>
<td>7.008</td>
<td>1.898</td>
<td>3.69</td>
<td>.001</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>compensating</td>
<td>11.94</td>
<td>9.115</td>
<td>3.025</td>
<td>3.01</td>
<td>.005</td>
<td>*</td>
</tr>
</tbody>
</table>

* calculated with 29 observations

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Figure 4-1: COMPENSATING AND EQUIVALENT VARIATION
Mean Bids With Risk Included
Figure 4-2: COMPENSATING AND EQUIVALENT VARIATION WITH RISK Dollar Differences

Risk of Aquifer Depletion in Percent

Differences Within Mean Bids

Type of Variation

- Equivalent
- Compensating
FIGURE 4-3: COMPENSATING AND EQUIVALENT VARIATION
Mean Bids Without Outlier
4.4 Deriving a Decision Weight Function

Introduction

This segment of the empirical analysis is concerned with the building of a decision weight function. It is designed to provide insight into the way individuals' dollar values change as probabilities of acquifer depletion or risk levels are varied.

This function is developed in an attempt to illuminate three areas. They are: (1) to find out if a linear and equivalent relationship exists between decision weights and their respective risk levels i.e., find out if all decisions have an equal decision weight; (2) if a nonlinear relationship is found, to find out how the respective probabilities are weighted; and (3) to compare the derived value curve derived from this study with the hypothetical value curve in Kahneman and Tversky [1984].

Relationship between Bids and Risk

In checking for linearity, mean bids are calculated for equivalent and compensating variation at each probability level. To derive a decision weight function, the percentage change between mean bids from one risk level to another have to be calculated. The percentage changes are calculated in the following manner. As an example: to derive the percentage change from 10 percent to 20 percent the mean bid figure at the 20 percent risk level is divided by the mean bid figure at the 10 percent level. This percentage is then plotted at the 20 percent risk
level. This method is used for all of the cases.

Originally, the mean bids associated with a zero percent chance of aquifer depletion were to be taken from questions numbered four and eight. These two questions refer to compensating and equivalent variation without the component of risk. As previously noted, the value for equivalent variation with a 10 percent level of risk is less than the value without any risk. The discrepancy between these bids led to the exclusion of the case which would have been derived from the risk level being changed to 10 percent from zero percent. Thus, the first point which is graphed in figure 4-4 is the percentage change in dollar bids when risk is varied from 10 to 20 percent.

If linearity is present between the decision weights, the graphing of these percentage changes with their respective risk levels would lead to a shape which is a reasonable facsimile of an indifference curve. The first derivative of acquifer depletion with respect to changes in bid levels should be negative throughout the function. The second derivative should be positive i.e., the function should be convex to the origin. In looking at figure 4-4 and table 4-3, the graph and the data do not indicate convexity. Figure 4-4 shows that the functions behave in a cyclical fashion.

The values in Table 4-3 are calculated in the following manner. The assumption was made that monetary outcomes are an expression for utility. Thus, for compensating variation, $19.75 is a measure of the total utility associated with a 100 percent chance of acquifer
depletion. If a linear relation associated with 10 percentage point increments was present, the dollar value of a 40 percent chance of aquifer depletion would be $7.90. This linear relationship was not found with any of the risk levels for either compensating or equivalent variation.

Probability Weights

In this section the slope of the decision weight function will be explored. Of particular interest is the way different probabilities are weighted. This is because of their affect on the value of the outcomes. For example: if the probability of losing well water is 10 percent, the value of Rattlesnake Creek water depends upon the weighting function. If the probability function is underweighted relative to the linear relationship which is dictated by decision theory, then the outcome becomes undervalued. However, if the probability function is overweighted, the value of the outcome is overvalued.

The previous section presented empirical evidence which showed that this probability function was not linear. How are the probabilities weighted? A probability function was built to answer this question. Remember that the slope of the decision weight function can be perceived as a measure of the sensitivity of preferences to changes in probabilities.
Table 4-3: The Amount by which Bids are Underweighted or Overweighted

<table>
<thead>
<tr>
<th>Level of Risk %</th>
<th>Level of Risk</th>
<th>Level of Actual (x) Total</th>
<th>Level of Actual (x) Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bids $</td>
<td>Utility Difference</td>
</tr>
<tr>
<td>10</td>
<td>4.52</td>
<td>2.89</td>
<td>1.63</td>
</tr>
<tr>
<td>20</td>
<td>6.40</td>
<td>5.77</td>
<td>.63</td>
</tr>
<tr>
<td>30</td>
<td>8.96</td>
<td>8.66</td>
<td>.30</td>
</tr>
<tr>
<td>40</td>
<td>10.71</td>
<td>11.54</td>
<td>-.83</td>
</tr>
<tr>
<td>50</td>
<td>13.54</td>
<td>14.43</td>
<td>-.89</td>
</tr>
<tr>
<td>60</td>
<td>15.10</td>
<td>17.32</td>
<td>-.22</td>
</tr>
<tr>
<td>70</td>
<td>16.93</td>
<td>20.20</td>
<td>-3.27</td>
</tr>
<tr>
<td>80</td>
<td>18.95</td>
<td>23.09</td>
<td>-4.14</td>
</tr>
<tr>
<td>90</td>
<td>21.88</td>
<td>25.98</td>
<td>-4.10</td>
</tr>
<tr>
<td>100</td>
<td>28.86</td>
<td>28.86</td>
<td>19.75</td>
</tr>
</tbody>
</table>

Note: If the difference between the actual bid and the risk multiplied by total utility is positive, the relevant probability or risk level is overweighted. If the difference is negative, the relevant probability or risk level is underweighted. As long as there is a difference, either positive or negative, there is the implication of nonlinear decision weights.
Figure 4–4: COMPENSATING AND EQUIVALENT VARIATION WITH RISK
Changes in Mean Bids

Type of Variation
- Equivalent
- Compensating

Risk of Acquifer Depletion in Percent

Percentage Change Between Bid Levels
The Weighting Function

Using expected utility theory, if the utility of having no well water is 'X' and the probability of aquifer depletion is 'P', then the value (for a monetary outcome) is \[X \times P\]. In this empirical analysis, the dollar representation of utility for 100 percent chance of no well water is $28.86 and $19.75 for equivalent and compensating variation, respectively. Thus, according to expected utility analysis, a 10 percent chance of losing well water should result in a utility representation of $2.88 using the equivalent variation figure [Kahneman and Tversky 1984 p.344]. This line of reasoning is followed for all of the risk levels. The points which are calculated in the above manner form a linear line with equal weighting for all probabilities.

Plotted against this line are the actual compensating and equivalent mean bids which were derived from the questionnaire. These plots can be viewed in figures 4-5 and 4-6. In figure 4-5, the empirically derived equivalent variation bids are overweighted for 10 and 20 percent. They are approximately equal at the 30 percent risk level. Bids which are associated with a 40 percent or greater chance of aquifer depletion are underweighted.

Figure 4-6 shows a similar analysis for compensating variation. However, bids are overweighted only at 10 percent, approximately equal at 20 percent, and underweighted for risk levels 30 percent or greater.
FIGURE 4-5:
COMPARISON BETWEEN EQUIVALENT VARIATION AND LINEAR DECISION WEIGHT FUNCTION

Type of Variation
- Equivalent
- Linear Function
FIGURE 4-6:
COMPARISON BETWEEN COMPENSATING VARIATION AND LINEAR DECISION WEIGHT FUNCTION

Risk of Acquifer Depletion in Percent

Dollar Bids

Type of Variation
- Compensating
- Linear Function

Risk of Acquifer Depletion in Percent

Dollar Bids
These plots indicate that for the most part, an increase of 10 percentage points in the probability of aquifer depletion raises the utility (in monetary terms) for Rattlesnake Creek by less than 10 percent. In terms of equivalent variation this implies: for each 10 percentage point increase in the probability of aquifer depletion greater than 30 percent, individuals demand less than a 10 percent decrease in their water bills in order to forego the use of Rattlesnake water. In terms of compensating variation: for each 10 percentage point increase in the probability of aquifer depletion greater than 20 percent, individuals are willing to pay less than 10 percent to be able to consume Rattlesnake Creek water.

4.5 Analysis of Explanatory Variables

The intent of the regression analysis portion of this study was to regress the dependent variables WTP and WTS upon the independent variables RESIDENC, RATTLEH2, Y, FUTURE, PREFERNCE, WATER, UTILITYS, and YEARS. This was to be done so that an explanatory equation concerning what the WTS and WTP bids were based upon, could be built. The relationship between the questions from the questionnaire and these variable labels is given in table 4-4.

After completing the initial regression analyses, it was found that the assumption of homoscedasticity was not satisfied. The term homoscedasticity relates to the assumption which states: the variances of each error term associated with all explanatory variables are
constant. If this assumption is not satisfied, the offending error terms are referred to as heteroscedastic or not constant.

Table 4-4: Variable Labels and their Respective Questions

| Question 1  | YEARS          | Question 7 = UTILITYS |
| Question 2 = RATTLEH2 | Question 8 = WTS |
| Question 3 = WATER      | Question 9 = RWTS |
| Question 4 = WTP        | Question 10 = RWTP|
| Question 5 = FUTURE     | Question 11 = Y  |
| Question 6 = FRAME      | Question 12 = PREFERENCES |

On a priori grounds it was expected that this study would have a small amount of heteroscedasticity. This is because the data which was collected for this study is cross sectional. It has been shown that there is a high incidence of heteroscedasticity with cross sectional data [Koutsoyiannis 1977 p.183].

There are many different techniques that can be used to detect heteroscedasticity. Perhaps the easiest technique is to plot the residuals against the predicted values. If heteroscedasticity is present, the plot will behave in either one or both of the following ways. The plots will disperse at an increasing rate as the predicted values increase, or, the plots will congregate together at an increasing rate. These descriptions refer to increasing variance and decreasing variance, respectively.
For this study, the WTP and WTS variables were regressed upon the independent variables. These predictive values were then plotted against the residuals. The analyses involving both dependent variables showed an increasing variance. This confirms that heteroscedasticity is a problem in this analysis.

There are several adverse consequences if the assumption of homoscedasticity is violated. One is that the regression method of ordinary least squares becomes inefficient. Efficiency refers to the property of having minimum variance around the unbiased estimators. Another problem pertains to the applicability of common second-order tests of the econometric assumptions. The second-order tests consist of analyzing the estimates of the parameters for their reliability. With the presence of heteroscedasticity, these tests become inapplicable [Koutsoyiannis 1977 p.184].

When heteroscedasticity is found to be present, the appropriate solution is to transform the offending estimates in such a way that these transformed estimates error terms have constant variance. A common transformation is to divide through the original relationship by the square root of the term responsible for the homoscedasticity [Koutsoyiannis 1977 p.188]. of multiplying through the original relationship by the natural log.

In this study, five out of the eight independent variables are dummy variables. A problem occurs with the transformation process. The problem is that dummy variables can not be transformed. The variables
WTP, WTS, WATER, UTILITIES, and YEARS were multiplied by the natural log in an attempt to derive constant variances. After running another set of regression analyses, the plots showed that the problem of heteroscedasticity had been taken care of. Although there was an R2 of 0.70, only one out of the eight independent variables was significantly different from zero. WATER was the significant variable. This variable was regressed by itself. This resulted in a very low R2.

Factor Analysis

Because of the problems associated with the use of regression analysis, factor analysis was chosen as a surrogate technique. Factor analysis is a multivariate statistical tool. Its function is to take variables with a high degree of intercorrelation which are difficult to interpret, and create new hypothetical variables called factors that are relatively independent and easier to interpret. See appendix B for a more in depth explanation of factor analysis.

The goal in factor analysis is to have each factor describe the variation shared in common by the subset of variables highly related to it and not describing the variation in the other variables. This is accomplished when there are high factor loadings on the highly related variables and very low factor loadings on the remaining variables. In this study, factor analysis is utilized to help quantify the meaningful factors which describe the independent variables. These are the same variables that were going to be the regressors in the regression analysis.
Stastical Methodology

In completing this factor analysis, the methodological steps as outlined in Kleinbaum and Kupper [1976] are followed. The first step consists of setting up the data for input. This involves imputing various combinations of communalities and, after completing steps two and three, finding the combination which produces the best results. Three different combinations of communalities are tried. They are: 1) use the communalities which are derived from the initial factor analysis. These are reinserted into the diagonal of the data matrix and the analysis is run again. This is repeated until the sets of communality values converge. 2) Use the largest (in absolute value) correlation coefficient associated with each variable. 3) use the original 1's which are associated with the major diagonal of the correlation matrix.

The second step involves determining the factors. There are many extraction techniques which can be used. The extraction methods called principal components and principal axis factoring are used in this study.

The final step involves the rotation of initial factors. This helps in the interpretation of the factors. There are two main methods of rotation. They are referred to as orthogonal and oblique rotation. There are several different techniques of rotation which fall under each of these main methods. For this analysis, varimax and quartimax were used, along with oblimin. The first two techniques fall under the
heading of orthogonal rotation while the latter belongs to the oblique method of rotation.

Results

After running factor analysis through the 18 possible combinations, the best results are derived using the extraction method of principal components, the quartimax method of matrix rotation, and having 1's on the diagonal correlation matrix. The results are in table 4-5. These numbers are from the rotated matrix.

In analyzing these results remember that an objective of factor analysis is to have each factor uniquely define a group of correlated variables. This is accomplished except for the variables Y and PREFERENCE. In these two cases, a significant proportion of their variation is accounted for in more than one factor.

Factor one explains 24 percent of the total variation in the data. To do this, it takes on the characteristics that the variables WATER, UTILITIES, Y, and RESIDENC have in common. A measure of wealth is an attribute that all of these variables have in common. As a result, factor one may be interpreted as a proxy measure of wealth.
Table 4-5: Results From Factor Analysis

<table>
<thead>
<tr>
<th>Variables</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>.7555</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utilitys</td>
<td>.7070</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>.5809</td>
<td>.4601</td>
<td></td>
</tr>
<tr>
<td>Residenc</td>
<td>.5304</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years</td>
<td></td>
<td>-.7456</td>
<td>-.4171</td>
</tr>
<tr>
<td>Prefrnce</td>
<td></td>
<td>.6569</td>
<td>.7354</td>
</tr>
<tr>
<td>Future</td>
<td></td>
<td></td>
<td>.6519</td>
</tr>
<tr>
<td>Rattleh2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Eigen Value | 1.919 | 1.326 | 1.268 |
% of Variation | 24 | 16.6 | 15.9 |

Factor two explains 16.6 percent of the total variation in the data. It subsumes the common characteristics of the variables YEARS and PREFRNCE. Note that there is a negative relationship between YEARS and factor two. Factor two reflects the attributes of feelings toward consuming Rattlesnake Creek water. Because of these attitudes, factor two is referred to as tastes. It was noted during the questionnaire process the reluctance of elderly people to pay more for any type of water. The major reason for this is budgetary constraints. Fixed incomes are prevalent among this age group. The existence of the variable Y in this factor further supports the hypothesis that factor two represents a viewpoint concerning the consumption of Rattlesnake Creek drinking water.
Factor three represents the combined effects that are unique to the variables' FUTURE and RATTLEH2, and to some degree PREFERENCE. This factor explains 15.9 percent of the variation within the data. A common embodiment between these variables is the expression of preference. Factor three is referred to as preconceived preferences. This is because the variables FUTURE and RATTLEH2 measure preconceived value judgements.

Several observations may be drawn from this analysis. Since 56.4 percent of the total variation in the data is explained by the three factors, there is a large amount of information which is unique to the individual variables. This implies that they all contribute some information to the study. A converse viewpoint may also be taken. If the three factors are able to explain only 56 percent of the variation, new variables should be added to the questionnaire. Hopefully, this would include more attributes which would be correlated with existing variables. This would result in a more complete explanation of the variation within the data. This analysis has shown the importance the three factors wealth, taste, and preconceived preference play in determining consumer's surplus values. If the goal of a study is to develop an explanatory equation of consumer's surplus, these three variables should be included.
CHAPTER V

IMPLICATIONS OF STATISTICAL ANALYSIS

5.1 Introduction

The three objectives of this paper as put forth in the preceding chapter are analyzed in the following sections. The dollar estimates for Rattlesnake Creek water are discussed in section 5.2. The problem of framing bias is analyzed in section 5.3. Section 5.4 contains the analysis pertaining to decision weights. Section 5.5 summarizes the results of this paper. It analyzes the effect that prospect theory has on the disparity between the two measures of value. The recommendations for areas of analysis which would be worthy of further study are also included.

5.2 Empirical Estimates for Rattlesnake Water

Discrepancies Between Compensating and Equivalent Variation

This section is concerned with the type of relationship that was found to exist between compensating and equivalent variation. It also calculates the consumers' surplus for Rattlesnake Creek water. The problems associated with this calculation are also explored.
The two measures of consumer's surplus, compensating and equivalent variation, have been shown to be significantly different from zero and significantly different from each other. In the Rattlesnake Creek study, the equivalent variation value is $6.40 and the compensation variation value is $2.80. This is a difference of $3.60, or equivalent variation is approximately 2.3 times larger than compensating variation. This difference is incompatible with present day economic theory. However, it does fall well within the range of size differentials of many other empirical analyses which have attempted to measure these values.

Sections 5.3 and 5.4 of this chapter will provide insight into why the difference between the measures of value exists. Before proceeding to these sections, an example is presented which illustrates the problem that occurs in an empirical study when compensating and equivalent variation differ.

Consumers' surplus for Rattlesnake Creek Water

To measure the excess value of Rattlesnake Creek water above its market price is to measure the consumer's surplus for Rattlesnake Creek water. To obtain the consumers' surplus figure, the total number of residential consumers of water is multiplied by either equivalent or compensating variation. In conversations with Mountain Water Company it was found that at any given time the number of water consumers is 16,000 with an approximate error margin of 200 consumers in either direction. This number includes flat rate and meter customers. The 16,000 figure
will be used for computational ease. The problem is choosing which measure of consumers' surplus should be used. If the compensating variation figure of 2.80 is chosen, the consumers' surplus associated with drinking Rattlesnake Creek water is $44,800. But, using the equivalent variation value of $6.40, the measure of consumers' surplus is increased to $102,400. It should be noted that these measures of consumers' surplus are very basic. For example, option value was not considered.

The implications for policy decisions are readily evident. The $50,000 annuity per year for the cost of a new water treatment plant is a good example. If the compensating variation value is used as the basis of consumers' surplus, the project is not feasible. However, the project becomes feasible when the equivalent variation calculation is used. This is assuming that either compensating or equivalent variation are the sole benefits of the project.

The question remains; which is the correct value to use? It has been suggested in the literature that the proper recourse depends upon the placement of property rights. According to Krutilla and Fisher [1975], if the individual owns the commodity in question, the proper measure is equivalent variation. If the individual does not own the commodity in question, then compensating variation is the proper measure.
There appears to be no concrete path to follow. Hammack and Brown [1974] used compensating variation in their study. This was because their analysis involved the study and preservation of waterfowl wetlands and the large majority of breeding sites that are threatened lie on private farmland. Thus, they used the criteria of property rights. However, Schulze et al. [1981] state that the disparity between the two measures of value is of little importance. They used both values.

Another technique which has been subscribed to in similar studies and the one which this paper will adhere to, is to use the two measures of value as boundaries. This is a conservative approach. It involves calculating the compensating value and using it as the lowest measure of consumer's surplus possible. The equivalent variation value is used as an upper boundary on consumer's surplus. Using this criteria, it is correct to state that the minimum aggregate consumers' surplus for Rattlesnake Creek water is $44,800. A positive aspect of using this approach is its conservatism. The low dollar value is hard to dispute.

5.3 The Question of Framing Bias

The purpose of this and the following section is to evaluate the empirical analysis to see if we may discern why compensating and equivalent variation differ. The hypothesis that framing bias has an impact upon the difference between compensating and equivalent variation is analyzed in this section.
Comparison of Questions and Values

In chapter four, it was established that the framing bias question elicited a mean bid of $4.09 which is statistically significant from zero. The next steps are to ascertain if there is any framing bias, and if so, its implications.

It is important to understand how the three questions concerning WTP, FRAME, and WTS were designed or framed. The WTP question asks the consumer to pay. In return, the individual gains the use of Rattlesnake drinking water. The FRAME question also asks the consumer to pay, but, the individual is asked to pay to avoid the loss of Rattlesnake drinking water. The WTS question is framed so that the individual receives payment which compensates him for the loss of drinking water. In summary, the questions concerning WTP and FRAME are similar in that the individuals are asked to make payments, and the questions involving FRAME and WTS are concerned with a potential loss to the individual.

In all of the following analysis, no outliers are excluded. It is felt that the outliers are not of a large enough distance from the mean values to warrant discarding them. The hypothesis that there is no difference between the mean bids of WTP and FRAME has been rejected at a high confidence level. This is interpreted to mean that framing bias is present in this study. Both WTP and FRAME measure a compensating variation value. The only difference is that the former is framed as a gain while the latter is framed as a loss. Thus, it was expected that there would be no difference between the two bids.
The presence of framing bias makes the hypothesis of invariance suspect. It appears that values can be manipulated by merely framing a question in a different perspective. This suggests that the problem of measurement concerning compensating and equivalent variation lies not in economic theory but rather in decision theory.

To further study the roles that perceived gains and losses play in decision theory let us turn the analysis toward the relationship between the questions concerned with WTS and FRAME. Both of these questions are framed so that the individual is confronted with a potential loss of a commodity. The null hypothesis of no difference between the means is accepted at the 88 percent confidence level. At this confidence level no conclusions can be made with certainty. It does suggest that there is a strong possibility that the means are related to one another in some manner. It can be stated that these two questions have a stronger relationship between each other than do the questions WTP and FRAME.

Conclusions

The following are the important observations which can be drawn from this section: 1) Framing bias is present and does have an impact on the dollar values which people give in response to direct questions; 2) The credibility of the hypothesis invariance is suspect; and 3) It appears that the way a question is framed has a greater impact on the answer than does the fact that a person makes or receives payments. More explicitly, making or receiving monetary payments affects the bid by a smaller amount than does the direction a decision is perceived to
be i.e., a gain or a loss.

It is also observed that the two questions associated with the loss of Rattlesnake water have higher dollar bids. This is in agreement with the loss aversion principle which was mentioned in chapter three. This loss aversion effects individuals regardless of whether they are making or receiving monetary payments.

The validity of the invariance hypothesis is brought into question by the preceding results. This is because of the existence of framing bias. The validity of the invariance hypothesis will continue to be explored in the following section. However, instead of framing bias, decision weights which are associated with various levels of risk will be the tools used.

5.4 Decision Weights

This section is concerned with the comparison of equivalent and compensating variation mean bids at various risk levels. The implications of the nonlinear decision weight function will be included. The impact which risk has upon the various bids will also be discussed. The effect of nonlinear decision weights upon the hypothesis of invariance is of particular interest.
Analysis of Values with Risk

It is shown in chapter four that at all ten risk levels, the hypothesis of compensating and equivalent variation values being equal is rejected by at least the 90 percent level. This confidence interval increased to the 99 percent level after the ten percent chance of acquifer depletion. Even though these variables differ significantly, they do share some similar traits.

The most interesting similar trait is the way the two values maintain their pattern of disparity throughout the bidding process. For example, the compensating mean bid for the ten percent risk of acquifer depletion is $3.15 and at 100 percent risk the mean bid is $19.75, a difference of 6.27 times. The equivalent mean bid for the ten percent level of risk is $4.52 and at 100 percent risk the mean bid is $28.86, a difference of 6.38 times.

Another example in which the two measures of value mimic each other may be seen in table 5-1. Notice how the increases in bids follow a pattern up to the 70 percent level of risk. The values alternate between an increase and a decrease over their previous bid values.

I feel that there are three possible explanations for the way the compensating and equivalent variation values react in similar patterns, even though their respective bid values do differ significantly from one another. The first reason is the small size of the sample. The answers which were elicited for the RWTS question were calculated using 29 observations. The second reason can be attributed to bidding bias. The
respondents in this survey may have placed a bid on one value based upon their bid on the other measure of value. For example: the respondents compensating variation value may be based upon their bid on the equivalent variation value. The compensating variation value may be a function of the equivalent variation.

The third explanation involves the assumption that the respondents reacted in a prescribed decision making process, perhaps in accordance with prospect theory. The data supports the hypothesis of loss aversion. At the ten percent level of risk, the equivalent variation value is 1.46 times greater than the compensating variation value. At the 100 percent level, the equivalent variation value is 1.43 times greater than the compensating value. This implies that when the individuals were confronted with the two value questions, there was a 'constant' factor which differentiated them. It is possible that this 'constant' factor is merely the measure for loss aversion or the compensation which is used to equilibrate a potential loss and gain situation.

Any one of these proposed solutions may be correct, or an entirely different hypothesis may provide the correct answer to this problem. This area may be worthwhile to explore in further study. A more complex questionnaire is needed in order to check for bidding bias or the possibility of a loss aversion factor.
### Table 5-1: Increases in Bids With Various Risk Levels

<table>
<thead>
<tr>
<th>Level of Risk</th>
<th>Compensating Increases Bids ($)</th>
<th>Equivalent Increases Bids ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3.15</td>
<td>4.52</td>
</tr>
<tr>
<td>20</td>
<td>3.97</td>
<td>6.40</td>
</tr>
<tr>
<td>30</td>
<td>5.39</td>
<td>8.96</td>
</tr>
<tr>
<td>40</td>
<td>6.42</td>
<td>10.71</td>
</tr>
<tr>
<td>50</td>
<td>8.19</td>
<td>13.54</td>
</tr>
<tr>
<td>60</td>
<td>9.45</td>
<td>15.10</td>
</tr>
<tr>
<td>70</td>
<td>10.27</td>
<td>16.93</td>
</tr>
<tr>
<td>80</td>
<td>11.94</td>
<td>18.95</td>
</tr>
<tr>
<td>90</td>
<td>14.68</td>
<td>21.88</td>
</tr>
<tr>
<td>100</td>
<td>19.75</td>
<td>28.86</td>
</tr>
</tbody>
</table>

---

**Analysis of Decision Weights**

The idea behind building a decision weight function is to determine if there is any merit in the hypothesis that risk plays a role in the disparity between compensating and equivalent variation. The theory of expected utility stipulates that the value of an outcome is a linear function of the probability of it occurring. Kahneman and Tversky put forth the hypothesis that the value of an outcome is not a linear function. They derived a weighting function which is dependent upon probability but not in a linear fashion. In accordance with Kahneman and Tversky's thesis, the decision weight function derived in chapter five is nonlinear.
The nonlinear shapes of the decision weight functions are very similar to the hypothetical weighting function which was derived by Kahneman and Tversky [1984]. Their hypothetical curve has the following similar characteristics: 1) Moderate and high probabilities are underweighted relative to sure outcomes; and 2) Low probabilities are overweighted and very low probabilities are usually overweighted quite highly.

Underweighting of medium to high probabilities contributes to risk aversion in gains by reducing the attractiveness of positive gambles. It also contributes to risk seeking in losses by lessening the aversiveness of negative gambles. The opposite effects are true in regard to the overweighting of low probabilities. People are risk seeking in events that are characterized by improbable gains and risk averse in situations which involve unlikely losses [Kahneman and Tversky, 1984].

Given these assumptions, what are the effects on the values of Rattlesnake water? In regard to the equivalent variation question, respondents reacted in a risk averse manner for the 10 and 20 percent probabilities and in a risk seeking manner for probabilities greater than 40 percent. Respondents reacted in the opposite manner when confronted with the question concerning compensating variation values. At the 10 percent level of risk, individuals gave a risk seeking response; for probabilities greater than 30 percent respondents gave a risk averse response.
There is one other matter to consider concerning decision weights; the effect of nonlinear decision weights upon the hypothesis of invariance. If a person reacts in a different manner to the same type of question, then the hypothesis of invariance is invalidated. This occurs when decision weights are nonlinear. For example, individuals responded in a risk averse manner when faced with moderate to high probabilities of success and in a risk seeking manner when faced with low probabilities of success (in the case of compensating variation). People reacted in different ways to the same question. This shows that the hypothesis of invariance is not valid due to the nonlinearity of decision weights. Kahneman and Tversky state that the invalidation of the hypothesis of invariance due to nonlinear decision weights..., "has been confirmed with both real and hypothetical monetary pay offs, with human lives as outcomes, and with a nonsequential representation of the chance process" [Kahneman and Tversky 1984 p.345].

5.5 Conclusions of Study

The Invariance Hypothesis

This paper was undertaken in an attempt to solve the question of why compensating variation and equivalent variation differ when empirically measured. It was proposed that a hypothesis within decision theory - invariance - does not accurately reflect the way individuals make decisions. Prospect theory was presented as an alternative method of describing the way in which individuals make decisions.
This study has provided a specific case where people do not react in a manner consistent with current economic theory. For example, the switching of decisions from being risk averse to risk seeking just by changing the level of risk is not consistent with economic theory. A new theory to explain decision making among economic agents may be necessary.

An empirical analysis involving Rattlesnake Creek water was designed. This analysis is similar to other studies that have been completed and which involve the measurement of compensating and equivalent variation. One difference between this and the other studies is the specific testing of the validity of the invariance hypothesis. Through the existence of framing bias and nonlinear decision weights, this study has shown that the invariance hypothesis is invalid. It is the author's hypothesis that this failure of invariance leads to the discrepancy between the two measures of consumer's surplus.

The hypothesis of invariance has been shown to be incorrect. People seem to react differently (in terms of higher or lower bids) depending on whether the decision they are confronted with is perceived as a loss or as a gain. Prospect theory differentiates between risky decisions which involve gains and risky decisions which involve losses. A hypothesis of prospect theory, the endowment effect, stipulates that a risky opportunity involving a perceived potential loss is valued more highly than is an equivalent opportunity involving a perceived potential gain. One of the objectives of this paper was to apply the hypothesis of endowment effects to the disparity problem between the two measures
of value. It is assumed that asking an individual to forego the use of a commodity in lieu of a payment is viewed as a loss of the good. Conversely, asking an individual to pay to obtain the use of a commodity is viewed as a gain to the consumer. This is because the individual has gained access to the good. Given the above assumptions; prospect theory provides a good explanation as to why the two measures, compensating and equivalent variation, differ. In this study, equivalent variation is associated with the gain of a good and compensating variation is associated with the loss of a good.

It is my hypothesis that the failure of invariance leads to the discrepancy between the two measures of consumer's surplus. The failure of the invariance hypothesis implies that individuals make decisions based on the way the problem is framed. Using the assumption that individuals differentiate between a decision which involves a gain and one that involves a loss, the disparity in measurements between compensating and equivalent variation may be explained. This disparity is based upon the loss aversion effect. People demand a larger payment when they feel that they are confronted with a loss.

This study indicates the disparity between compensating and equivalent variation is inevitable so long as decisions are based upon the loss aversion principle and the hypothesis of invariance is not followed. The conclusions stated above indicate that the decision theory which is currently the state of the art be evaluated and altered so that it can accommodate the preferences and attitudes which this study has shown individuals to have.
Prospect Theory and its Potential

It has been shown that prospect theory deviates from the decision theory which is currently employed in economic theory. It is beyond the resources of this study to test the validity of prospect theory. However, at the level which this paper has been able to explore prospect theory, the results appear to explain some of the discrepancies which are associated with the measurement of compensating and equivalent variation. Similar to the examples which were given in chapter two, the major concepts will be illustrated. However, in these examples the empirical study will be the reference point.

It has been shown that the hypothesis of decision weights is viable. However, this study was not designed to test the impact that decision weights have on the outcomes which were derived. The only conclusions that can be drawn about decision weights from this study is that, depending upon the probability of an event occurring, they result in underweighting and overweighting of outcomes.

The concepts associated with the value curve, framing bias, and consumer's surplus were tested in this paper. The question eliciting the compensating variation value was framed as a potential gain of Rattlesnake Creek water and the question eliciting the equivalent variation value was framed as a potential loss of Rattlesnake Creek water. According to the hypotheses which underly the value curve and in particular the loss aversion hypothesis, the question which is concerned with the loss should result in a higher value. The equivalent variation

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measure was greater. The testing of framing bias also resulted in accord with decision theory. It was found that objectively equivalent questions derived different results. Both questions required the respondents to pay a sum of money. The only difference was that in the WTP question the individual would be acquiring Rattlesnake Creek water and in the FRAME question the individual would be avoiding the loss of Rattlesnake water. In accord with the loss aversion effect, the question associated with the potential loss - FRAME - elicited the higher value. Thus, the major concepts of decision theory all predicted correctly in this empirical study.

Impact of This Study

This study has proven that people do not react in a manner which is recognized by current economic theory. For example, the switching of decisions from being risk averse to risk seeking just by changing the level of risk is not consistent with economic theory. A new theory to explain decision making among economic agents may be necessary. Prospect theory has been presented, not necessarily as an alternative theory but rather as one which incorporates actual attributes of decision makers.

Although there are problems found within the decision theory which is incorporated in microeconomics, the conclusions of this paper do not dictate the revamping of microeconomic theory. However, this study does point out that there is a need to reconcile reality i.e., individuals making actual market decisions and microeconomic theory. The author
suggests that this reconciliation be brought about by alleviating the deficiency which has been shown in decision theory. Primarily, this study has proven that the concepts of decision weights, framing bias, and loss aversion should be included in a revamped decision theory.

Areas of Further Study

There are four areas of research which I believe would be beneficial to pursue. All of these will help rectify the disparity between compensating and equivalent variation.

One area of study involves the realm of psychology. This analysis would determine how individuals make decisions under risk. It would be of significance to find out if decisions are based upon a cognitive decision making process, a spur of the moment decision, or something entirely unrelated.

One of the major problems associated with the empirical study presented in this paper is the small sample size. To validate the results which are presented in this paper, many additional studies are needed. In particular, these studies should concentrate their objectives on determining if the relationship holds between gains versus losses, and payments made versus payments received. The idea that a perceived loss is weighted more heavily in an individual's decision making than an equivalent cash outlay is a new concept which requires more proof.
Another interesting area for further research lies in the direction of decision weights. It would be of interest to find out what causes the weighting function to be weighted differently at various levels of risk. In this study, decision weights alternated between high and low values for most of the function. Finding out if this is a consistent pattern of behavior, and if so, why, would be valuable information.

A fourth area of research would be the derivation of individual utility functions. The utility functions would be derived for two reasons. The first reason would be to determine if individuals behave in both a risk seeking and a risk averse manner. If this proves to be correct, the next step would be to analyze the respective slopes of the utility function. It would be interesting to see if a relationship could be found between the risk averse and risk seeking portions of the functions and the disparity between the compensating and equivalent variation values.
APPENDIX A

Questionnaire Used For Data Collection

This appendix contains the questionnaire which was derived to elicit responses concerning Rattlesnake Creek drinking water. Respondents were read the background information and the questions. No additional information was given until after the interview. It is assumed that this minimizes information bias. Also listed in this appendix are tables which contain the data that was used in the statistical analysis.

Questionnaire:

Type of residence: Apt. 1 room__, 2 or greater__. House, less than 5 rooms__, 5 or greater__.

INTRODUCTION

Until 1983, a large portion of Missoula consumed Rattlesnake water for approximately nine months out of every year. Since the problem of Giardia developed in the summer of 1983, Missoula water consumers have relied completely on well water.

Some problems have been noticed by consumers since they have started using well water on a full time basis. One of the problems is that Missoula's well water is seven times as hard as Rattlesnake water. Hardness refers to the amount of mineral content in the water. The harder water results in deposits building up in appliances such as dish washers. Restaurants complain about the difficulty of having their dishes look clean. For home owners this problem often necessitates the installation of a water softener.
There is a positive health effect which is attributed to harder water. Medical evidence shows that there is improvement within the cardio vascular system when hard water is consumed.

At present there are studies underway which are analyzing the feasibility of installing a water treatment plant on the Rattlesnake. If a treatment plant was installed it would make Rattlesnake water available for personal consumption once again. I would like to emphasize that this study is in no way concerned with the current problem involving ownership of the water company.

1) How many years have you lived in Missoula? ____.

2) Have you previously consumed Rattlesnake water? 0=No ___, 1=Yes ___.

3) What is your average water bill per month? $_______.

4) Assume that it would be possible to consume Rattlesnake water if a water treatment plant is built. What is the maximum amount which you would be willing to have added on to to have added on to your existing monthly water bill to give you the opportunity to be able to consume Rattlesnake rather than well water? ________.

NOTE: IF ALL UTILITIES ARE SUBSUMED UNDER RENTAL AGREEMENT ASK: what is the maximum amount you would be willing to have added on to your monthly rental agreement?

5) Do you believe that it will be possible for you to be able to consume Rattlesnake water in the foreseeable future? 0=NO _____, 1=YES ____.

6) Imagine that you are currently consuming Rattlesnake water and the water company issues a report which states that they will be forced to shut off Rattlesnake water next month due to high costs unless consumers would be willing to pay higher water bills. What is the maximum increase in your monthly water bill that you would allow in order that you continue the Rattlesnake water service? ____________.

7) Excluding the cost of your water bill, what would your total utility bill be per month on the average? $________.

8) Assume that a law is passed which outlaws the building of a water treatment plant on the Rattlesnake. This means that the Rattlesnake source would be unavailable for usage for the foreseeable future. What is the minimum amount which could be deducted from your monthly water bill that would just compensate you for not being able to use Rattlesnake water? $_______.

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In the following set of questions I am going to test to see how your monetary valuations change as I vary the probabilities of an event occurring. I am going to present to you different decisions which involve various levels of risk or probability. These decisions are concerned with aquifer depletion. Many people believe that this problem exists today. An aquifer can be thought of as an underground lake which is fed by very slow moving underground streams. Our well systems withdraw water from these lakes. Aquifer depletion occurs when the wells are withdrawing more water from the lake than can be replenished by the feeder streams.

Note: Ask the respondent each percentage.

9) Assume that you know the chance of Missoula's underground lakes being completely depleted within the next five years is:

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but, by consuming Rattlesnake water this problem would be partially if not fully alleviated. If using well water creates some negative feelings for you, how much would have to be paid through lower bills to continue to use only well water.

10) If you knew that by continuing to consume only well water the risk of depleting Missoula's aquifer was:

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what is the maximum amount you would be willing to pay through increases in your monthly water bills to be able to consume Rattlesnake water?

11) Which one of the following categories would the income level of this household belong to:

Category 1: $0 - 8,000. ____.

Category 2: $8,000 - 16,000. ____.
Category 3: $16,000 - 24,000. ___.
Category 4: $24,000 - 32,000. ___.
Category 5: $32,000 - 40,000. ___.
Category 6: $40,000 - 50,000. ___.
Category 7: Greater than $50,000 ____.

12) Which type of water would you prefer, Rattlesnake or well water, given that both cost the same and were equally safe? 0=Well ___, 1=Rattle ____.

The variable names are listed below, along with the appropriate questions which pertain to them. The variable RESIDENC is not associated with any question. It was noted by the individual giving the survey.

Question 1 = YEARS
Question 2 = RATTLEH2
Question 3 = WATER
Question 4 = WTP
Question 5 = FUTURE
Question 6 = FRAME
Question 7 = UTILITYS
Question 8 = WTS
Question 9 = RWTS
Question 10 = RWTP
Question 11 = Y
Question 12 = PREFRNCE
### Data Elicited From Survey

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Comments Pertaining To the Data

The negative numbers in the data set indicate missing values. A (-1) means that the respondent refused to answer the question. A (-7) indicates that the respondent draws water from a private well. A (-8) indicates that the respondent had either their utility and/or their water bill included within their rent payments. The values (####) indicate that the respondent refused to answer the question. There are more non-responses associated with questions nine and ten than any other questions. This is because of the high degree of complexity associated with them. In particular, it was hard for individuals to respond to the hypothetical scenario that was presented to them.

In looking at the data one observes several numbers that appear with a high frequency. For example the number 10.98. This number is a common dollar figure for a water bill. Respondents apparently used this figure as a reference point for answering some of the other questions.
APPENDIX B

Factor Analysis

To fully explain the concept and the working mechanics of factor analysis would require a paper devoted entirely to this subject. This appendix is an overview of the concept of factor analysis plus a glossary of the terms which were mentioned in the central body of the paper. In addition to the glossary, the implications of the terms, (when appropriate), are given.

The objective in factor analysis is to create new variables which uniquely define two or more intercorrelated variables. This enhances data interpretation and alleviates multicollinearity problems in regression analysis by using the newly created variables. In factor analysis: "each variable may be regarded as a dependent variable that is regressed on a set of independent 'unobserved' factors, each of which in turn is a function of all original variables" [Cooper 1983 p.145].

Communalities: This term refers to diagonal elements on the correlation matrix. Obviously, these numbers are equal to 1 on an untouched correlation matrix. However, some researchers believe that it is correct to replace these superfluous 1's with some other numbers (called communalities) and then subject this revamped correlation matrix to factor analysis [Kleinbaum and Kupper 1976]. This study used the extraction method called principal axis and replaced the diagonals with
communalities for one analysis. These communalities consisted of the largest (in absolute value) correlation for each value [Kleinbaum and Kupper 1976 p.386]. This did not prove to be productive.

Eigen Value: This is the total amount of variance for all of the variables which is accounted by each factor. It is calculated by squaring and then summing all of the factor loadings for each factor. With the eigen value it is easy to calculate the percentage of total variation explained by each factor. In factor analysis the variables are standardized. Thus, the percentage of explained total variation is calculated by dividing the eigen value by the total number of variables used in the analysis.

Extraction method: Refers to the technique which is used to generate the factors. Principal components analysis and principal axis factoring are the two extraction methods used in this study.

Factor: This term refers to the hypothetical variables created using factor analysis. A factor can be defined as a weighted linear combination of the original variables where the weights are estimated from the data. [Kleinbaum and Kupper. 1976 p.381] Positive attributes of factors consist of interdependence among factors and either very high or very low factor loadings among individual factors. In this study there are three factors that are derived. They are referred to as wealth, tastes, and preconceived preferences. These names apply to factor one, factor two, and factor three, respectively. The factors are referred to in this manner because they reflect these values.
Factor loadings: These are coefficients which describe the correlation between a derived factor and an original variable. These coefficients point out how each variable is correlated to each factor under optimum conditions. It is preferred that each variable has a high factor loading associated with only one factor, with the rest of the loadings being very small.

Principal components: This is a technique used to derive factors. With this method, no assumptions about the underlying structure of the variables i.e., collinearity, are made. This technique seeks: "that set of linear combinations of the original variables that absorb and account for the maximum possible proportion of total variation in those variables" [Cooper 1983 p.144]. This technique: Determines factors in such a way so as to explain as much of the total variation in the data as possible with as few of these factors as possible" [Kleinbaum and Kupper 1976 p.389]. The principal components method of extraction is the method used which derives the most satisfactory results.

Principal axis factoring: This extraction technique is equivalent to principal components as far as underlying assumptions and intent of analysis. The only difference lies in its computational ability. It allows for the use of communalities within the analysis.

Rotation: The initial matrix resulting from factor analysis consists of the original variables and the new factors. Quite often these initial results are difficult to interpret. Rotation is a term which describes a manipulation of the factor matrix which enables the
results to be perceived more clearly. Rotation is used, "if it is desired to find meaningful underlying factors that describe the variation in a set of variables" [Kleinbaum and Kupper 1976 p.391].

There are two types of rotation methods: orthogonal and oblique. In orthogonal rotation the axis of the factors are rotated at 90 degree angles from each other. These axes are rotated until the factors have the highest and/or lowest factor loadings attainable. The oblique method of rotation is very similar to the orthogonal method. The main difference lies in the way the axes are rotated. There is no precise relationship held between the axes. They rotate to whatever pattern gives the best results. Varimax and quartimax are deviations from one another and belong under the heading of the orthogonal method while oblimin is a type of oblique method.

Unlike its counterpart oblique rotation, factors which are derived using the orthogonal method are statistically uncorrelated. However, the oblique method often results in a more clear relationship between the original variables and the factors. This frequently results in a tradeoff in choosing between independence or clarity. Fortunately, in this study quartimax rotation has the best clarity or structure as well as being independent among the factors.
APPENDIX C

Historical Development of Consumer's Surplus

Jules Dupuit was the first person to work with the concept of consumer's surplus. Dupuit was interested in how to measure public utility. He stated that the price which society pays for a public good is a minimum measure of the utility that society derives from the project [Dupuit 1844 p.256]. Dupuit felt that there were many examples of when the value of a public good (to the public) was substantially greater than the price which was paid to acquire the good. From this idea, Dupuit brought forth the concept of consumer's surplus.

After Dupuit's work, the theory of consumer's surplus lay dormant for over fifty years. It was revived by Alfred Marshall in his seminal work Principles of Economics. Marshall defined the term consumer's surplus to be: "The excess of the price which he, (the consumer), would be willing to pay rather than go without the thing, over that which he actually does pay" [Marshall 1922 p.124].

As a good proxy for the measurement of consumer's surplus, Marshall put forth the idea of using the area under the relevant portion of the demand curve. This idea has passed the stringent test of time. But, because of the assumptions which underly his demand curve (known as the Marshallian demand curve), his analysis has been revamped. Before explaining the theoretical problems which are associated with the Marshallian demand curve it is helpful to describe the relationship
between consumer's surplus and the demand curve.

The Relation Between Consumer's Surplus and a Demand Curve

Concepts that are fundamental to consumer's surplus are the demand curve and the law of demand. The definition of a Marshallian demand curve is: the various quantities per unit of time that consumer's are willing to buy at all possible price alternatives, holding constant tastes and preferences, price of other goods, and nominal income. Assuming the marginal utility of money was constant, Marshall used the law of diminishing marginal utility to deduce the shape of the demand curve. Marshall's analysis took the following line of thought. Since the marginal utility of money is constant, the ratio between the marginal utility of a commodity and its price is a constant ratio. This results in the downward slope of the demand curve. For example, if the price of good X decreases, the marginal utility of the good must also decrease in order to maintain the same ratio. By the law of diminishing marginal utility, this is accomplished by increasing the quantity demanded. Therefore, Marshall's law of demand stipulates that price and quantity have an inverse relationship with no exceptions. More succinctly stated, utility is maximized when the ratios of marginal utilities of all commodities and income are equal to their price ratios.

Marshall assumed that the price a consumer is willing to pay for a good is a cardinal measure of the marginal utility or the marginal benefit which the consumer will realize upon the purchase of the commodity. If the price which is available to the consumer is greater
than what he deems as his marginal benefit, he will not complete the transaction. If the price just equals the marginal benefit of the good, the consumer will make the transaction. In this and the former scenario there is no positive consumer's surplus that accrues to the buyer.

When the purchase price of a good is less than the amount that the consumer is willing to pay, the term consumer's surplus comes into context. Obviously, if the market price for an additional unit of the product is less than what a particular person is willing to pay, there is some benefit which the consumer attributes to the good above the value which the market has placed upon the good. This "extra" value is the surplus which the person acquires from the acquisition of the product. Because of Marshall's universal statement concerning the law of demand, there will always be positive consumer's surplus so long as the market price is less than what the buyer is willing to pay.

There is an alternative way of thinking of the relationship between the market price and the price a consumer is willing to pay. Due to the law of diminishing marginal utility and the rational maximization principle, a consumer places a higher value on the first unit of a good (reflected in a higher paid price), than he does for the second unit. The same idea holds with the third, fourth, etc., units. Thus a demand curve is a locus of marginal valuations of different levels of output. So long as the price is less than the consumer's marginal valuation for the first unit, the consumer will attain some added value — consumer's surplus.
Problems in Marshall's Analysis

Marshall's analysis has given ammunition to the detractors of consumer's surplus. As I have previously explained, the demand curve and consumer's surplus are related. Therefore, the assumptions in Marshall's theory which relate to the demand curve affect the validity and/or the accuracy of this concept. The two main problems of his analysis as it pertained to consumer's surplus were his failure to recognize income effects and his use of cardinality in utility theory. It should be noted that all economic theorists of the 19th century employed cardinal utility in their analyses.

The use of cardinality in utility theory implies that it is possible to impute a number which corresponds to a level of satisfaction which is induced by the purchase of a good. To determine the quantities of goods which an individual will buy at given prices, Marshall's theory assumes that one must know his utility surface. The utility surface purports to state by how much one commodity bundle is preferred or desired over another. Marshall believed that it was possible to derive numerical values from a utility surface which would give exact measures of utility.

Marshall's second error concerns disregarding income effects and their effect upon demand. Marshall was able to ignore income by making the assumption that the marginal utility of money was constant. As Marshall phrased it: "At one and the same time, a person's material resources being unchanged, the marginal utility of money to him is a
fixed quantity, so that the prices he is just willing to pay for the two commodities are to one another in the same ratio as the utility of those two commodities" [Marshall 1922 p.95]. This means that the effect on demand, because of changes in real income resulting from changing prices, is neglected. This implies that changes in nominal income will not affect the marginal rate of substitution between any good and income. Therefore, if the consumer's income decreases and the price of the good remains constant, the price will still be equal to the marginal rate of substitution, without any change in the amount of the good purchased. Thus, the consumer's demand for any commodity is independent of their income [Hicks 1946]. Apparently, Marshall felt that most goods purchased are, by themselves, a negligible percentage of total income. This would mean that any substitution effects which are present would overwhelm any income effects, making their analysis irrelevant.

These two errors on Marshall's part cast doubt upon his analysis involving demand curves. After Marshall, there was little substantial improvement in the development of consumer's surplus until Hicks. Hicks provided the theoretical tools which clarified the concept of consumer's surplus.

Hicks' Theory

Sir John R. Hicks played an eminent role in the development of demand theory. He used the foundation which Marshall laid, but differed in his treatment of demand in many areas. Two of the more prominent aspects which he revamped are cardinality and nominal income.
The assumption of cardinal utility theory is very restrictive if not unrealistic. A much less restrictive assumption, that of ordinal or preference theory, was put forth by Pareto in 1909. With this assumption, the consumer is only required to be able to rank his preferences for purchases. The magnitude of the rankings are irrelevant. This concept expunged a serious pitfall in economic literature. Unfortunately, after putting forth the idea of ordinal utility, Pareto took it no farther.

In papers involving Allen and Hicks [1934], Slutsky [see Allen 1936], and Hicks [1939], preference theory was incorporated with demand theory. In Hicks' *Value and Capital* he purged all concepts tainted with cardinal utility. He then proceeded to rebuild consumer's demand theory using scales of preferences. Instead of using the notion of cardinality, Hicks measured amounts of money or the amount of money spent on the (n-1) consumption goods as a measure of welfare. This was known as Hicks' composite commodity theorem. This theorem was an exposition simplification. It allowed a group of commodities to be analyzed as a single good, so long as their price ratios remained constant throughout the analysis [Hicks 1956 p.36]. This made two dimensional analysis possible.

Marshall's demand theory, when reinterpreted using the new definition of utility, had greater strength due to the less restrictive assumption, while keeping the theory intact. A more troublesome aspect of Marshall's theory as it pertains to consumer's surplus is his lack of treatment of income effects that are associated with a price change. In
Marshall's *Principles* he deduced the downward slope of the demand curve. There is an inverse relationship that must always hold between price and quantity demanded. He states: "The one universal rule to which the demand curve conforms is that it is inclined negatively throughout the whole of its length" [Marshall 1922 p.99]. Marshall implies that the income effect of a price change is negligible. By making the marginal utility of money constant it enabled him to ignore the changes in real income which come about because of a price change. By keeping the marginal utility of money constant, a price change, which changes income, does not affect the consumer's demand for the good because his demand for any commodity is independent of income.

Income Effects

Hicks realized that income effects could have a significant effect upon a demand curve. It should be noted that there are other interpretations of Marshall besides that of Hicks. Milton Friedman [1949] interpreted Marshall as assuming that nominal income remained constant over the entire demand curve, but, at different prices real income changed. However, the prevalent theory has followed Hicks'.

Using two different methods, Hicks analyzed how a price change affected a consumer. In *Value and Capital* Hicks used an indifference diagram to illustrate substitution and income effects. In *A Revision of Demand Theory* Hicks used a marginal valuation curve as well as an indifference diagram to illustrate the two effects. He came to similar conclusions using both techniques. His analysis using indifference
curves will be illustrated below.

In Allen and Hicks [1934], an important concept referred to as the expenditure curve (since renamed the income-consumption curve) was used. Income changes which result from price changes and their affect upon quantity demanded are illustrated by the income consumption curve. It is derived in the following fashion.

Take an indifference mapping of two goods A and B and let the locus of points between F and G signify the price line or budget constraint of the goods (see figure A3-1). Now, a new price line JK which is parallel to the old one is drawn. Since JK is parallel to FG, the relative prices of the two goods have not changed. What has changed is income. It has been raised. This results in a higher level of utility, illustrated by I1. Now, if a line is drawn between the consumption points W and Z, we have an income-consumption curve. Holding their relative prices constant, it shows how the demand for the two goods are affected by changes in income [Hicks 1946].

Hicks discovered that there are income effects that are associated with a price change. For a price decrease: "it makes the consumer better off, it raises his real income, and its effect along this channel is similar to that of an increase in income" [Hicks 1942 p.31].
FIGURE A3-1
Income Consumption Curve
Just how much does this income effect change the Marshallian demand curve? The consensus in the literature is that it depends upon the amount of money the consumer spends on the good in relation with his total income. Quoting Hicks in Value and Capital: "For the extent to which he is made better off by a fall in the price of X will depend upon the amount of X which he was initially buying; if that amount was large relatively to his income, he will be made much better off, and the income effect will be very important; but if the amount is small, the gain is small, and the income effect is likely to be swamped by the substitution effect" [Hicks 1942 p.32]. Marshall was apparently only concerned with the latter, taken to its extreme because he had the income elasticity of demand equal to zero.

Depending upon the income elasticity of demand for the good, the income effects may act in two ways. If the good is normal, the income effect will complement the substitution effect. It is also possible for the good to be inferior, meaning that the income effect counteracts to some degree the substitution effect. Leaving out this income effect could result in a poor estimate of the magnitude of consumer's surplus due to incorrectly shaped demand curves.

Substitution Effects

Substitution effects can be measured in a manner which is analogous to the technique used for the income effects (see figure A3-2). However, when substitution effects are being measured, income is held constant and the price of one commodity is changed. The original price
line is ML. The price of B is then lowered with the subsequent price line being MP. Because of the law of diminishing marginal rate of substitution, a price decrease for good (B) will always result in an increase in the quantity demanded, W to Z.

The substitution effect can be proven to behave in a certain manner because of the law of diminishing rate of substitution. Hicks found that the substitution effect is always inverse to the direction of the price change. This is in agreement with Marshall and has no effect upon the theory of demand.

The law of demand can now be seen to be a sum of two components: the substitution and income effects which arise from a price change. A fall in the price of a good raises the consumer's real income, and this effect upon demand is similar to that of an increase in income. A fall in the price also changes relative prices, resulting in the substitution effect. As long as the good in question is normal or the good does not take up a large percentage of income, the demand curve will always be inversely related to price.

Income Compensated Demand Curves

After Hicks presented the idea that income effects could have an effect upon a demand curve, he proceeded to develop a demand curve which took income effects into account. He called this an income compensated demand curve. Basically, he adjusted the Marshallian demand curve so as to allow for the effects of the changes in real income as different positions on the demand curve are taken up.
FIGURE A3-2
Substitution Effects Curve
FIGURE A3-3
Ordinary and Compensated Demand Curves
There are several additional assumptions in conjunction with Marshall's analysis which underlie this demand curve. The first assumption dictates the presence of a weak preference ordering system. The assumption of weak preference ordering allows us to show that the consumption position which is chosen on the income compensated demand curve is preferred to any position which lies below the demand curve [Hicks 1956 p.43]. The assumptions underlying the income compensated curve mandate that the original amount consumed (A) must be indifferent to another amount consumed (B) which lies upon the same curve. This indifference is achieved through continually adjusting income so that the consumer is no better off at either position A or B. Since the consumer's income is continually adjusted for price changes or, the consumer's real income is held constant, the movement along the income compensated demand curve must only measure the substitution effects of price changes. Since substitution effects are always inversely related to price changes, so long as non-satiation holds, this demand curve must always slope downward.

A second assumption is that the generalized commodity M is available in amounts finely divisible. The third assumption, which I alluded to previously, pertains to the non-satiation of M. The consumer must always prefer more M to a lesser amount, given the consumption of X remains unchanged [Hicks 1956].
Figure A3-3 shows an ordinary Marshallian demand curve AB and an income compensated demand curve Ab for a normal good. Marshall's curve lies outside that of Hicks'. This illustrates that for a price change from P to P1, the consumer has an additional amount of spending power associated with Marshall's curve. This is shown by the difference between the quantities of X purchased at P1. Hicks' income compensated demand curve lies below Marshall's curve because all points on Hicks' curve have had real income adjusted.

The portions of demand theory that relate to the theory of consumer's surplus that go beyond the above points are presented in the main text. The Hicksian theory of demand is what the most current concepts concerning consumer's surplus are based upon. With a firm comprehension of this theory, the problems which currently beset consumer's surplus and the suggestions which are presented in this paper will be more readily understood.
BIBLIOGRAPHY


