Digital filtering and modelling of the gravity field of the Bitterroot Valley western Montana

Kenneth J. Wells

The University of Montana

1989

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DIGITAL FILTERING AND MODELLING OF THE GRAVITY FIELD OF THE BITTERROOT VALLEY, WESTERN MONTANA

By
Kenneth J. Wells
B. A., University of Montana, 1984

Presented in partial fulfillment of the requirements for the degree of Master of Science University of Montana 1989

Approved by:

Chairman, Board of Examiners

Dean, Graduate School

Date May 19, 1989

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Previous gravity studies of the Bitterroot valley were based on simplifying assumptions about the density of valley fill and the smoothness of bedrock topography. This study uses density measurements of drill cores and known depths to bedrock from drill holes to provide greater control for modelling on a computer.

I used previously published Bouguer anomaly values from various authors. I assembled the data into a regularly spaced grid using a computer, then computed the Fourier coefficients of the grid with a fast Fourier transform (FFT) computer algorithm. The FFT coefficients were used for filtering in the frequency domain. In order to separate residual and regional gravity anomalies, I analyzed results from several band-pass and vertical continuation filter operations. Bedrock cross-sections were constructed with inverse and forward modelling, using residual gravity profiles.

The anomaly profiles and cross-sections were examined for evidence of faulting along the Stevensville and Tin Cup lineaments, two northeast trending fault sets which may bound a down-dropping block within the Bitterroot valley. No evidence was found for faulted bedrock in the valley along these two lineaments.
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I must also thank my other committee members, Dick Lane and Nancy Hinman, for their patience and promptness in reviewing this document. In particular, I thank Nancy for her willingness to replace an earlier committee member and for tips on how to get things done. Without Dick’s help in familiarizing me with the arcane details of FORTRAN and Unix in the early stages of this project, its completion would have been impossible.

Thanks for financial support from the Montana Power Company, and Mrs. Jeanette Waltari of Great Falls. Thanks for inspiration from my colleagues Josef Crepeau and Luther Strayer. Thanks to the faculty and staff at the geology department for a quality education.
1. Introduction

The principal objectives of this study are to interpret and integrate existing information about the geology and gravity of the Bitterroot valley in order to create structural cross-sections showing bedrock depth. Two subsidiary objectives are to look for evidence of the Stevensville and Tin Cup lineaments in the cross-sections, and to establish a computing environment at the University of Montana Geology Department for future potential field studies.

The gravity data used in this study are from two sources: a compilation by the Department of Defense on digital tape, and a doctoral dissertation by Lankston (1975). Bouguer anomalies and their locations are listed in appendix D. Depth to bedrock in the valley is known from four drill logs compiled by Abramiuk (1980). Bendix Field Engineering Corporation drilled five deep (300 - 700 m) holes in the Bitterroot valley in 1978 and 1979 as part of a United States Department of Energy inventory of uranium resources. Rock types and specific gravities are known from drill cores from four of the five holes (Table 1). Information about the structure and geology of the Bitterroot valley used in this study is compiled from various authors.

With completed cross-sections I investigated the hypothesis advanced by Cartier (1984) and Barkmann (1984) that the Bitterroot valley floor between Hamilton and Stevensville, Montana is dropping relative to the rest of the valley along two northeast trending faults, and found that the lineaments are not associated with vertical displacements in bedrock.
By using more data and more advanced gridding and filtering methods, I also improved on the detail of Lankston's (1975) gravity study of the Bitterroot valley, and found evidence for an area of rugged bedrock topography which he had discounted. The computer programs I use in this study will simplify future investigations into gravity and magnetic fields and be useful for future potential field studies conducted at the University of Montana.

The computer programs I used for gridding (Webring, 1981), computation of Fourier transforms (Hildenbrand, 1983), and inverse modelling (Webring, 1985) were provided by the United States Geological Survey office in Denver, Colorado. Where necessary I translated code so that the programs now run on UNIX mainframe and workstation computers at the University of Montana. The code can be obtained on 1/2 or 1/4 inch magnetic tape from the Geology Department, University of Montana, Missoula MT 59812.

I used these programs and existing data to produce structural cross-sections. The irregularly spaced field data are interpolated into evenly spaced grids with minimum curvature by a cubic spline sheet algorithm (Briggs, 1974), then the Fourier transforms of the interpolated grids are calculated for use in digital filtering. After filtering, performing the inverse Fourier transform, and separating regional and residual gravity, I selected west-to-east profiles across the valley for inverse and forward modelling.
FIGURE 1

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2. Bitterroot Valley Regional Bedrock, Valley Fill, and Their Densities

The Bitterroot valley trends 100 km south from Lolo, Montana to Conner, Montana (Figure 1). The valley averages about 16 km from east to west. The average gradient of the Bitterroot River decreases from 3.8 m/km between Conner and Hamilton, to 1.9 m/km from Hamilton to Lolo (McMurtrey et al., 1972). The drainage area of the Bitterroot basin is approximately 7300 km² (Cartier, 1984). To the west, the Bitterroot valley is bounded by the Cretaceous and early Tertiary granitic rocks and mylonites of the Idaho-Bitterroot batholith which form the Bitterroot Range (Figure 2). The valley is bounded to the east by Precambrian sedimentary and metasedimentary rocks, Cretaceous and early Tertiary granitic and ultramafic intrusive rocks, Miocene volcanic rocks, and associated metamorphic rocks which make up the Sapphire Range (Lindgren, 1904; McMurtrey et al., 1972; Snyder, 1985).

Almost all of the Bitterroot Range and much of the Sapphire Range are granitic rocks. The late Cretaceous Idaho-Bitterroot batholith is mostly medium-grained muscovite-biotite granite with a granodiorite border zone (Hyndman, 1983). There are also small anorthosite bodies within the granitic batholith (McMurtrey et al., 1972). Lindgren (1904) noted that "in the Bitterroot Mountains streaks of basic magmatic segregations often appear". Foster (1986) found that these mafic dikes are andesitic to tonalitic, are on the average 2.4 m thick, and may comprise 20% or more of the Idaho-Bitterroot batholith's volume. I measured the specific gravity of four andesitic samples from a dike near Weir Creek, Idaho, in the

pC = Precambrian sedimentary and metasedimentary rocks of the Belt Supergroup.

IBb = Cretaceous to early Tertiary granitic and mylonitic rocks of the Idaho-Bitterroot batholith.

Tv = Tertiary volcanic rocks.

Ts = Tertiary sedimentary rocks (Six Mile Creek formation equivalents).

Q = Quaternary sediments, fluvial and glacial.

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Idaho-Bitterroot batholith. Their mean density is 2.85 gm/cm³. I also measured 17 granitic samples from Weir Creek and from the Bendix drill cores. Their mean density is 2.60 (see Table 2). If mafic dikes comprise 20% of the batholith's volume the overall density of the batholith is 2.65 gm/cm³.

Granitic gneiss, averaging over 600 m thick, is exposed along the western margin of the Bitterroot valley (McMurtrey et al., 1972). This gneiss, mylonitized to various degrees in different locations, is the border zone of the Idaho-Bitterroot batholith. Mylonites are also found on the southeastern, southern, and southwestern margins of the batholith (Garmezy, 1981). Hyndman and Myers (1988) suggest that there are two distinct east-dipping mylonites, both showing top-to-the-east movement, along the eastern margin of the Idaho-Bitterroot batholith:

1) a Paleocene granitoid amphibolite facies mylonite (A-mylonite),
2) an overlying late Eocene greenschist facies mylonite/chlorite breccia (G-mylonite).

Approximately 57 to 53 m.y.a. the A-mylonite formed within the Idaho-Bitterroot batholith during rapid east-west extensional thinning of the crust (Hyndman and Myers, 1988). Sediments began filling the Bitterroot valley at this time. The G-mylonite on the eastern flank of the Bitterroot Range formed along a listric normal fault about 40 m.y.a., most likely as the batholith rose isostatically (Garmezy, 1981; Hyndman and Myers, 1988).

Core samples from holes MB-6, MB-8, MB-9, and MB-12 show that gneiss and the G-mylonite are bedrock along the western margin of the
Bitterroot valley, and sheared granite is the bedrock of the valley floor (Figure 3). Bedrock on the eastern margin of the valley is gneiss, Cretaceous granite and some Precambrian sedimentary rocks (Abramiuk, 1980; McMurtrey et al., 1972). I measured the density of 14 mylonite and gneiss drill core samples using bedrock and conglomerate clasts. Their mean density is 2.57 gm/cm³ (see Table 2).

Precambrian sedimentary and metasedimentary rocks make up a small amount of the rocks of the Bitterroot valley and adjacent foothills. There are exposures along Lolo Creek in the north, in the foothills on the eastern margin of the valley, and at the head of the South Fork of the Bitterroot River in the south. The Precambrian rocks of the study area are mostly argillaceous limestones of the upper Prichard Formation, and quartzite, metaquartzite and argillites of the Ravalli group and Wallace Formation (McMurtrey et al., 1972).

Eocene epizonal andesites, rhyolites and ashflow tuffs are found on the eastern and southern margins of the Idaho-Bitterroot batholith (Hyndman, 1983), and in the foothills of the Sapphire range. Prominent exposures are west of Florence near the foothills of the Bitterroot Range (Lindgren, 1904).

In a previous investigation of the gravity field, Lankston (1975) states that the Bitterroot valley is filled in places with more than 1300 m of Tertiary sediments. The Bendix drill holes penetrate as much as 800 m. Four lithologic units comprise this Tertiary fill. Along the eastern and western valley margins, mylonite, gneiss and Precambrian sedimentary
FIGURE 3 Generalized geologic structure across N5162000, Bitterroot valley. After Abramiuk, 1980. Hole MB-9 projected 600 m south. This is the simplest model incorporating known depths. Fault on eastern margin of valley is not mapped, but expressed by Bouguer anomalies.

B = Precambrian sedimentary and metasedimentary rocks of the Belt Supergroup.
1Bb = Cretaceous to early Tertiary granitic rocks of the Idaho-Bitterroot batholith.
Su = undifferentiated upper Tertiary and Pleistocene conglomerates and sandstones.
S = Six Mile Creek formation equivalent conglomerates and sandstones.
Rm = mid-Tertiary conglomerates and sandstones.
R = Renova formation equivalent siltstones and sandstones.
rocks are overlain by late Eocene to early Oligocene cobble/boulder conglomerates. Clasts are granite, gneiss and quartzite in a fine sand or calcareous matrix. Towards the center of the valley these conglomerates interfinger with late Eocene to early Miocene Renova Formation equivalent sediments. In the center of the valley Renova equivalents overlie sheared granite (Figure 3). Renova equivalents in the Bitterroot valley are interbedded claystones, siltstones and sandstones with minor coal and conglomerate interbeds (Abramiuk, 1980; Norbeck, 1980). Early Miocene to early Pliocene cobble/boulder conglomerates and Six Mile Creek Formation equivalents unconformably overlie Renova equivalents. Six Mile Creek equivalents in the Bitterroot valley are conglomerates with granite, gneiss and sandstone clasts, micaceous sandstones, and minor claystone interbeds (Abramiuk, 1980).

In my models of the valley, I assign each of the four valley fill lithologic units a density based on specific gravity measurements of core samples and percentages of fine and coarse material recorded in the drill logs (Abramiuk, 1980). This realistic model of heterogeneous valley fill produces a more accurate bedrock depth than a model assuming one homogeneous fill unit. Lankston (1975) used a homogeneous valley fill model with a density contrast of -.50 gm/cm$^3$, as did Manghnani and Hower (1962). My measurements of densities of the different units are listed in Table 2.

There is an average of about 10 m of Quaternary sediments over the older rocks and sediments of the Bitterroot valley (McMurtrey et
al., 1972). Modern flood plain deposits of the Bitterroot River are typically underlain by late Pleistocene to Holocene gravels and sands (Abramiuk, 1980; McMurtrey et al., 1972). Presently the Bitterroot River is rapidly depositing bedload between Charles Heights and Stevensville (Cartier, 1984).
3. Faulting and Structural Trends

Numerous geologists have identified the four dominant trends (Figure 2) in the structural grain of the Bitterroot valley and surrounding area (Barkmann, 1984): these are north-south, northeast-southwest, northwest-southeast, and east-west. Strike-slip and dip-slip movements are seen in all four of the dominant orientations. The most prominent tectonic feature of the Bitterroot valley is the north-trending normal fault coincident with the mylonite zone on the eastern margin of the Bitterroot Range. This fault extends for over 100 km along the front of the Bitterroot range and Abramiuk (1980) suggests it dips about 20° to the east. The regional Bouguer anomaly of the study area is dominated by the effect of this normal fault and the density contrast along the western margin of the Sapphire Range (Figure 2).

McMurtrey et al. (1972) note that the traces of four northeast trending en-echelon folds are exposed along the front of the Bitterroot Range between Victor and Florence. Their axial surfaces strike from N35°E for the southernmost to N15°E for the northernmost, and dip more than 45° to the east. The Stevensville Lineament (Barkmann, 1984), which crosses the Bitterroot valley, is sub-parallel to the southernmost en-echelon fold. Surface evidence for faulting on the Stevensville Lineament includes:

1) slope breaks aligned with the lineament across high terraces,
2) abrupt changes in vegetation patterns across the lineament (perhaps due to disrupted groundwater flow),
3) a marked change in the behavior of the Bitterroot River where it crosses the lineament (Barkmann, 1984; Cartier, 1984).

Barkmann (1984) also identified the northeast trending Tin Cup Lineament just north of Charlos Heights. Faulting along the Tin Cup Lineament is shown at the surface by measured right-slip displacement of up to 2 km in breccias, and N60°E faults dipping 60° to 90° to the northwest exposed in road cuts near Como Lake. Movement along these exposed faults is right-slip and northwest-down. Faults trending N40°E are exposed where the Tin Cup Lineament, a system of Eocene strike-slip faults and Quaternary normal faults, crosses Skalkaho Creek in the Sapphire Range (Barkmann, 1984; Kuhns, 1980). Barkmann (1984) estimated the rate of motion of the block in the last century at approximately 3 cm/yr. Given sufficient density contrast and displacement to create a signal in the gravity field, such a block's bounding faults should be detectable by a gravity survey.
4. Methods and Results

I assumed gravity data submitted by different parties had the same accuracy and after careful checking found no conflicting values for overlapping observations. Lankston’s dissertation (1975) provided thorough coverage of the Bitterroot valley floor. Data from other investigators, compiled by Department of Defense on magnetic tape, covered the Sapphire Range thoroughly, and the Bitterroot Range much less completely.

The data I used for this study are published with coordinates of latitude and longitude. Unlike latitude and longitude, x-axis and y-axis of the Universal Transverse Mercator (UTM) projection have equal units (meters). I chose to transform coordinates into the UTM projection to simplify digital processing. The Fortran program I wrote to convert coordinates is listed in Appendix A.

Next, I processed the irregularly spaced data set into a regularly spaced grid. MINC (Webring, 1983), the program used for gridding in this study, uses a two-dimensional cubic spline algorithm which minimizes the total curvature of the grid and preserves original observations (see Appendix C). After trying several grid spacings I chose 500 m as the smallest spacing which results in a reliable grid with a minimum of aliasing. A sampling rate of once every 500 m results in a Nyquist frequency of one cycle per 1000 m (Strang, 1986). I roughly estimate the mean distance between gravity observations within the Bitterroot valley at 500 ± 200 m (Appendix D), so the mean Nyquist frequency of the original
data is also near one cycle per 1000 m. Any trends in the data with 
frequency higher than 1/1000 m (or wavelength less than 1000 m) will not 
be resolved accurately with this grid. Geologic features such as buried 
stream channels, clay lenses, or small faults will not be detected in the 
grid. Some small areas on the fringes of the study area were flagged by 
MINC as being too far from the nearest observation and assigned a flag 
value. These areas were filled in manually to approximate existing trends 
in the grid and fit published regional values for long wavelength data 
covering western Montana and east-central Idaho. This manual 
smoothing of the grid margins prevents the introduction of spurious high-
frequency anomalies to the grid. In no case are manually approximated 
values used as observed anomalies for inverse modelling.

Once the data were gridded, I used two programs for Fourier 
transforms and filtering: FFTFIL (Hildenbrand, 1983) and Pro-Matlab 
(Moler et al., 1987). Both incorporate a complex matrix algebra FFT (fast 
Fourier transform) algorithm (Strang, 1986) and perform various grid 
filtering operations in the frequency domain (see Appendix C). Pro-Matlab 
provides more flexibility in filter design and uses the full graphics 
capability of our Geology Department's SUN Microsystems 3/100 
workstation. For FFT computation and filtering, I extracted two 
64 × 64 grids with origins at N5138000 E711000 and N5106000 E711000 
from the larger grid (Figures 4-7). A 64 × 64 matrix is the largest on 
which Pro-Matlab can perform FFT computations.
FIGURE 4  Bouguer anomaly map of northern Bitterroot Valley. Contour intervals 2 mgal. Coordinates in km, UTM Zone 11T. Bold lines are bedrock outcrop. Scale 1:250000.
FIGURE 5 Rendered surface of observed Bouguer anomaly values, northern Bitterroot valley. View to northwest. X, Y coordinates in km.
FIGURE 6 Bouguer anomaly map of southern Bitterroot Valley. Contour intervals 2 mgal. Coordinates in km, UTM Zone 11T. Bold lines are bedrock outcrop. Scale 1:250000.
FIGURE 7  Rendered surface of observed Bouguer anomaly values, northern Bitterroot valley. View to northwest. X, Y coordinates in km.
I used various computer programs for forward and inverse modelling of gravity anomalies. Forward gravity modelling is the process of calculating the measurements which would result from a given geological model, comparing calculated with observed measurements, and adjusting the model, while inverse gravity modelling is the process of calculating a geological model from observed data (Dobrin, 1976). SAKI (Webring, 1985), the inverse modelling computer program I used, requires as input an initial model and a profile of observed gravity values. The program then computes improved models whose calculated gravity profiles match observed gravity more closely, until the root-mean square difference of the two profiles is within a user-specified tolerance. The process is actually a combination of forward and inverse modelling.

Gravity modelling requires reasonable constraints on the densities of the rocks in question. Fortunately some samples of Bitterroot valley fill are available at the University of Montana. In the late 1970's Bendix Field Engineering Corporation drilled eleven holes to bedrock in the Missoula and Bitterroot valleys as part of a Department of Energy survey of uranium resources of the United States. About eighty feet of split core from drill holes MB-6, MB-8, MB-9, and MB-12 in the Bitterroot valley (Figure 1) are stored at the University of Montana, thanks to the involvement of Dr. Robert Fields. No core was recovered from hole MB-11. To gather accurate data on at least a small portion of the subsurface density distribution I measured the specific gravity of 74 core samples. Measurements were made with a triple beam balance and a bucket of
water, using the formula:

\[
\text{Specific Gravity} = \frac{\text{Weight suspended in air}}{\text{Weight(in air) - Weight(in water)}}.
\]

Table I summarizes the specific gravity data. As discussed earlier, Abramiuk (1980) proposed a model of four stratigraphic units of Bitterroot valley fill:

- **Su** = undifferentiated upper Tertiary and Pleistocene conglomerates and sandstones;
- **S** = Six Mile Creek formation equivalent conglomerates and sandstones;
- **Rm** = mid-Tertiary conglomerates and sandstones;
- **R** = Renova formation equivalent siltstones and sandstones.

After measuring specific gravities of the core samples, I believe these densities can be assigned to polygons in a geologic model for inverse modelling (Figure 3).
Table 1.

<table>
<thead>
<tr>
<th>Hole</th>
<th>Footage</th>
<th>p</th>
<th>N</th>
<th>Rx Type</th>
<th>Rx Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MB-6</td>
<td>470-480</td>
<td>2.05</td>
<td>3</td>
<td>coarse ss*</td>
<td>Su</td>
</tr>
<tr>
<td>MB-6</td>
<td>470-480</td>
<td>2.47</td>
<td>2</td>
<td>granitic cobbles</td>
<td>Su</td>
</tr>
<tr>
<td>MB-6</td>
<td>619-629</td>
<td>2.05</td>
<td>3</td>
<td>medium/fine ss'</td>
<td>Su</td>
</tr>
<tr>
<td>MB-6</td>
<td>867-877</td>
<td>2.55</td>
<td>4</td>
<td>granite/gneiss cobble</td>
<td>Rm</td>
</tr>
<tr>
<td>MB-6</td>
<td>1092-1104</td>
<td>2.05</td>
<td>5</td>
<td>rounded sandy cgl*</td>
<td>Rm</td>
</tr>
<tr>
<td>MB-6</td>
<td>1092-1104</td>
<td>2.28</td>
<td>2</td>
<td>rounded coarse/med cgl</td>
<td>Rm</td>
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<td>3</td>
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<td>Rm</td>
</tr>
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<td>1</td>
<td>mica-rich ss matrix*</td>
<td>Rm</td>
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<td>2</td>
<td>granitic cobbles</td>
<td>Rm</td>
</tr>
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<td>MB-6</td>
<td>1727-1736</td>
<td>2.58</td>
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<td>greenschist mylonite</td>
<td>IBb</td>
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<tr>
<td>MB-8</td>
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<td>1.89</td>
<td>6</td>
<td>fine to coarse ss'</td>
<td>R</td>
</tr>
<tr>
<td>MB-8</td>
<td>2322-2325</td>
<td>2.09</td>
<td>4</td>
<td>fine angular cgl*</td>
<td>R</td>
</tr>
<tr>
<td>MB-8</td>
<td>2322-2325</td>
<td>1.91</td>
<td>1</td>
<td>fine ss'</td>
<td>R</td>
</tr>
<tr>
<td>MB-8</td>
<td>2709-2723</td>
<td>2.60</td>
<td>8</td>
<td>granitic basement</td>
<td>IBb</td>
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<tr>
<td>MB-9</td>
<td>970-980</td>
<td>2.51</td>
<td>8</td>
<td>granitic basement</td>
<td>IBb</td>
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<tr>
<td>MB-11</td>
<td>No core</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MB-12</td>
<td>768-778</td>
<td>2.12</td>
<td>5</td>
<td>coarse ss*</td>
<td>Rm</td>
</tr>
<tr>
<td>MB-12</td>
<td>768-778</td>
<td>2.60</td>
<td>2</td>
<td>gneiss cobbles</td>
<td>Rm</td>
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<td>880-892</td>
<td>2.01</td>
<td>3</td>
<td>v. fine ss*</td>
<td>Rm</td>
</tr>
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<td>880-892</td>
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<td>5</td>
<td>rounded medium cgl*</td>
<td>Rm</td>
</tr>
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<td>880-892</td>
<td>2.56</td>
<td>2</td>
<td>gneiss cobbles</td>
<td>Rm</td>
</tr>
</tbody>
</table>

*unconsolidated samples were lightly coated with paraffin

\( p = \text{density in gm/cc} ; N = \text{number of samples measured} \)
Summary logs describing chips from drill holes (Abramiuk, 1980) show percentages of lithic fragments ("conglomerate"), sand and clay for each ten foot interval.

Table 2.

<table>
<thead>
<tr>
<th>Hole</th>
<th>Interval</th>
<th>Frags</th>
<th>$\rho$</th>
<th>Fines</th>
<th>$\rho$</th>
<th>Avg $\rho$</th>
<th>Rx Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MB-6</td>
<td>0'-630'</td>
<td>81%</td>
<td>2.47</td>
<td>19%</td>
<td>2.05</td>
<td>2.39</td>
<td>Su</td>
</tr>
<tr>
<td>MB-6</td>
<td>630'-1570'</td>
<td>22%</td>
<td>2.58</td>
<td>78%</td>
<td>2.17</td>
<td>2.26</td>
<td>Rm</td>
</tr>
<tr>
<td>MB-8</td>
<td>0'-1200'</td>
<td>39%</td>
<td>2.47</td>
<td>61%</td>
<td>2.05</td>
<td>2.21</td>
<td>S</td>
</tr>
<tr>
<td>MB-8</td>
<td>1200'-2450'</td>
<td>15%</td>
<td>2.58</td>
<td>85%</td>
<td>1.96</td>
<td>2.05</td>
<td>R</td>
</tr>
<tr>
<td>MB-9</td>
<td>0'-100'</td>
<td>80%</td>
<td>2.47</td>
<td>20%</td>
<td>2.05</td>
<td>2.39</td>
<td>S</td>
</tr>
<tr>
<td>MB-9</td>
<td>100'-700</td>
<td>28%</td>
<td>2.58</td>
<td>72%</td>
<td>2.17</td>
<td>2.28</td>
<td>R</td>
</tr>
</tbody>
</table>

* no core; rho assumed same as in MB-6

$\rho$ = density in gm/cc ;

frags = percentage lithic fragments ;

fines = percentage finer than coarse sand ;

for explanation of rock units see Figure 3 .
To isolate the residual and regional Bouguer anomalies in the Bitterroot valley, I constructed two frequency-domain ramped band-pass filters. The low-pass filter removed all components with frequency higher than 1 cycle/16 km, and the high-pass filter passed frequencies higher than 1 cycle/16 km (Figures 8-15). I empirically chose 16 km, the average width of the Bitterroot valley, as the cutoff wavelength for band-pass filtering. Cutoffs at much shorter wavelengths retained anomalies of a more regional nature, flattening out the residual. Longer cutoff wavelengths up to 20 km have nearly the same results as 16 km. I used the low-pass filter to isolate the regional, and the high-pass to isolate the residual. The resulting regional anomaly grid is dominated by the normal fault on the west flank of the Bitterroot valley. The regional gravity field I constructed this way is very similar to an upward continuation of several hundred meters.

The regional trends I isolated are limited to the scale of the Bitterroot valley and surrounding mountain ranges, an area of roughly 10,000 km$^2$. On a larger scale, the regionals I used are compatible with what is known about the crustal structure of southwestern Montana. Information about crustal structure in this area comes from various seismic investigations (e.g: Sheriff and Stickney, 1984; Carlson, 1985; DeBoer, 1985; Clement, 1986). South of the Montana Lineament, which runs roughly through the Missoula valley (Figure 1), the crust is thinner and has a higher heat flow than to the north, with the crust/mantle
FIGURE 8  Low-pass filtered map of northern Bitterroot valley Bouguer anomalies.
Contour interval 2 mgals.
Cutoff = 16 km.
Scale 1:250000.
Coordinates in UTM projection, Zone 11T.

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FIGURE 9  Low-pass filtered rendered surface of Bouguer anomalies, northern Bitterroot valley.
Cutoff = 16 km.
FIGURE 10 High-pass filtered map of northern Bitterroot valley Bouguer anomalies.
Contour interval 2 mgals.
Cutoff = 16 km.
Scale 1:250000.
Coordinates in UTM projection, Zone 11T.
FIGURE 11 High-pass filtered rendered surface of Bouguer anomalies, northern Bètterroot valley.
Cutoff = 16 km
FIGURE 12 Low-pass filtered map of southern Bitterroot valley Bouguer anomalies. Contour interval 2 mgals. Cutoff = 16 km. Scale 1:250000. Coordinates in UTM projection, Zone 11T.
FIGURE 13  Low-pass filtered rendered surface of Bouguer anomalies, southern Bitterroot valley. Cutoff = 16 km
FIGURE 14 High-pass filtered map of southern Bitterroot valley Bouguer anomalies.
Contour interval 2 mgals.
Cutoff = 16 km.
Scale 1:250000.
Coordinates in UTM projection, Zone 11T.
FIGURE 15  High-pass filtered rendered surface of Bouguer anomalies, southern Bitterroot valley.
Cutoff = 16 km
interface dipping 3° to the northwest (Carlson, 1985; Clement, 1986). The Bitterroot valley's antiroot is a regional high in the mantle running through Missoula and Challis, Idaho, with crustal thickness decreasing from 33 km at Missoula to 29 km at Challis (Carlson, 1985). Ballard (1980) describes the regional gravity of southwest Montana with a low-order polynomial surface. This method does not accurately resolve crustal structure in the area of the Bitterroot valley.

I chose representative west to east anomaly profiles from the residual grids for inverse modelling. Density data from drill cores (Table 1) support a four rock unit, six-body model for valley fill (Figure 3). While inverse modelling, I noted that the total error of my models was not sensitive to small changes in the density of any of the bodies (± 0.025 gm/cm^3). The greatest standard deviation for density measurements of each rock unit is also about 0.025 gm/cm^3. My measurements (Table 2) show that the density contrast between bedrock and valley fill ranges between -0.71 and -0.32 gm/cm^3. I believe the density values I calculated from measuring cores are also representative of the uncored intervals. The four bodies against the valley walls are denser than the two in the center of the valley, due to the predominance of boulders and coarse conglomerates derived from the valley walls. The lower three bodies are less dense than the three upper bodies. Although Constenius (1988) models the Tertiary Kishenehn basin in northwest Montana with sediments increasing in density continuously with depth from the top to the bottom of the basin, this is not the case in the Bitterroot valley. The
climate and topography of the valley in the late Tertiary may have created a higher energy depositional environment, allowing deposition of larger clasts further toward the valley center.

Sediments within each lithologic unit of Bitterroot valley fill are probably also compacted and become denser with depth. However the core samples and drill logs show that denser units overlie less dense ones. This "sandwich" configuration of density contrasts, with the least dense layer in the middle, above crystalline bedrock may attenuate anomalies caused by faults passing through both density interfaces. For instance, a reverse fault in the bedrock/lower Tertiary interface will cause a negative anomaly of a certain amplitude. The same fault passing through the lower Tertiary/upper Tertiary interface will reverse the slope and lessen the amplitude of the anomaly (Figure 16). If density of valley fill did increase continuously with depth, vertical displacement would be easier to detect from Bouguer anomalies.

The shapes of the six density bodies in any particular cross-section are necessarily a simplification of the subsurface. In some profiles the depth to each density contrast is known at several points, from drill hole data. For others, the only constraints are bedrock outcrop locations. Section A-A' across N5162000 was the most difficult to resolve. This section has good control provided by holes MB-6 and MB-8. I also projected the depths to density interfaces southward from hole MB-9, after correcting for elevation difference and valley width. There are high-frequency fluctuations in this anomaly profile not found in those in the
FIGURE 16 Gravity anomalies over vertical faults with identical displacement.
Top: less dense layer overlies denser layer.
Bottom: Denser layer overlies less dense layer.
In some portions of the Bitterroot valley, denser valley fill overlies less dense fill, which in turn lies on denser bedrock. The additive gravity anomalies from the two separate density interfaces may attenuate the amplitude of the total gravity signal.
southern portions of the valley, where the gravity profiles are smooth (Figure 20). However the fluctuations extend north to south for several kilometers and I believe they represent bedrock topography. For section A-A', I modelled the interface between lower Tertiary and upper Tertiary rocks as relatively planar in comparison to the bedrock/valley fill interface. I could not get models in which the upper interface mimicked the lower one to converge to a geologically reasonable configuration with acceptably low root-mean square error. The possibility that the upper interface mimics the lower one cannot be ruled out since any faults more recent than late Tertiary will offset bedrock and valley fill. However inverse modelling does not provide any positive evidence for it. Figure 19 presents my model for profile A-A'.

Lankston (1975) initially indicated rugged bedrock topography a few kilometers south of my profile A-A' (along the line dividing T.9N and T.10N). He attributed this to inaccuracy in his computer algorithm and filtered out the high frequency components, to imply a smoother valley floor. After close examination, I found that the high-frequency fluctuations in Bouguer anomalies in this area between N5157000 and N5165000 are present in the original observations and are not signal processing artifacts. They represent bedrock offsets with amplitude up to 300 m and wavelength between 1000 m and 2000 m (Figure 18). The area between Stevensville and Florence, where the valley is widest, has the most rugged bedrock topography.
FIGURE 18 Residual Bouguer anomalies across N5162000.
FIGURE 19 Section A-A', bedrock model across NS162000. See Figure 3 for key to rock types.
FIGURE 21  Bedrock model B-B' across NS128000. See Figure 3 for key to rock units.
FIGURE 22  Calculated and observed residual gravity at N5138000.

$\triangle$ = calculated

--- = observed
FIGURE 23  Section C-C', bedrock model across N5138000. See Figure 3 for key to rock types.
FIGURE 24  Residual Bouguer
anomalies (in mgals) in the vicinity
of the Stevensville lineament.
Top: N5150000
Bottom: N5154000

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FIGURE 25  Residual Bouguer anomalies (in mgals) in the vicinity of the Stevensville lineament.*
Top : N5158000
Bottom : N5162000
5. Conclusions

To find evidence for the Stevensville and Tin Cup lineaments I examined Bouguer anomaly profiles (Figures 18,20,22), contour maps, and bedrock profiles generated by inverse modelling. Given my grid's Nyquist frequency of one cycle/1000 m, it was not possible in this study to resolve any displacements which might cause gravity anomalies with wavelengths less than 1000 m. I have only completed inverse modelling of three profiles, A-A' at N5162000, B-B' at N5128000, and C-C' at N5138000 (Figures 19,21,23). Neither the residual Bouguer anomaly profiles nor the bedrock models show evidence for the two lineaments. Barkmann (1984) proposes that the Stevensville Lineament crosses the Bitterroot valley between N5152000 and N5165000, and trends northeast (Figure 2). The anomaly profiles across the valley in this area are for the most part smooth, with some short wavelength, low amplitude fluctuations (Figures 24,25). Between N5158000 and N5164000 there are high amplitude (four to six mgal), short wavelength (1.5 to 4.0 km) fluctuations, trending due north, in the residual Bouguer anomalies. Model A-A' (Figure 19) across N5162000 shows the high topographic relief (300 to 600 m) of bedrock which causes these short wavelength anomalies.

The Tin Cup Lineament crosses the Bitterroot valley between N5105000 and N5115000, trending northeast (Figure 2). The residual Bouguer anomaly profiles and bedrock model B-B' (Figures 20,21) show no evidence for bedrock faulting within the valley along this lineament. There is evidence in anomaly profiles N5110000 and N5112000 for a
north-trending east-down normal fault (Figures 26,27). The upper three rock units of profile A-A' are only a thin veneer (less than 50 m thick) in profiles B-B' and C-C', so I did not include them in the models.

Profile C-C' (Figure 22) across N5138000 shows a smooth U-shaped gravity anomaly with no evidence for faulting within the valley. Bedrock model C-C' (Figure 23) shows that there is over 1100 m of fill in the center of the valley, with bedrock depth of 100 m below sea level.

There are three more specific conclusions about the geology of the Bitterroot valley which I can draw from my gravity study:

1) The lowest residual gravity values within the valley are found from N5130000 to N5140000 (Figures 10,14), so the 1100 m bedrock depth in profile C-C' is a good approximation of the maximum depth to bedrock in the valley.

2) The drill hole data from hole MB-6 at N5162000 and the configuration of bedrock profile A-A' (Figure 19) show that the G-mylonite on the western margin of the valley dips subvertically in that area. The other residual anomaly profiles show that it dips steeply all along the valley.

3) A steep-sided anomaly low, running north-south at E732000 between N5158000 and 5163000 (Figure 10), indicates an unmapped fault displacing bedrock in that area. This north-south zone also has the greatest bedrock relief and the most east-west extension in the valley.
Whichever tectonic conditions caused the valley to spread out almost twice as much between Stevensville and Florence as elsewhere presumably formed a full graben, rather than the half-graben configuration of the rest of the valley. Future gravity studies using finer grids over the Bitterroot valley between Stevensville and Florence could help explain why bedrock topography in that area has so much vertical relief compared to the rest of the valley. Additional field measurements of gravity are necessary to construct a reliable grid with finer spacing than I used. For example, gravity stations 50 m apart would resolve 100 m displacements on the Stevensville or Tin Cup lineaments. Magnetic measurements could also provide additional control for future investigations.
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7. Appendix A: Coordinate Conversion

Gravity data compiled by Department of Defense and the data in Lankston's (1975) dissertation have x,y coordinates in latitude and longitude. Since one degree of latitude does not equal one degree of longitude, it is necessary to convert coordinates into a geographic projection with equal units before gridding. The Universal Transverse Mercator (UTM) projection uses easting and northing units of kilometers, and is thus ideal for use with the computer programs used in this study for gridding and inversion.

The UTM projection divides the globe into 60 meridional zones each six degrees of longitude wide, with the principal meridian for zone 1 at 177° west. Polar zones north of 84° north and south of 80° south are excluded from the projection. In the northern hemisphere the origin of each zone is at the equator and 500,000 m west of the zone's principal meridian. To minimize average error throughout the zone the scale along the principal meridian is set to 0.9996 rather than 1.0 (Dozier, 1980). This concentrates the error inherent in any projection of spherical data onto a plane in the center of the zone rather than the margins. The study area straddles the boundary between UTM zones 11T and 12T. UTM projection allows overlap of zones, and I have assigned all data points to zone T11.
UTM northing (N) and easting (E') are calculated from the formulae:
\[ N = I + p^2[(II) + (IIIp^2)] + A_6 \quad \text{and} \quad E' = p[(IV) + (Vp^3)] + B_5 , \]

where \( I = S \times k_0 \);

\[ II = \frac{(v \sin\phi \cos\phi \sin^21^\circ)}{2 \times k_0 \times 10^6} ; \]
\[ III = \frac{(\sin^41^\circ \sin\phi \times \cos^3\phi)}{2 \times (5-\tan^2\phi+9e^{-2}\cos^2\phi+4e^{-4}\cos^4\phi) \times k_0 \times 10^6} ; \]
\[ IV = \frac{v \cos\phi \sin1^\circ \times k_0 \times 10^4}{} ; \]
\[ V = \frac{(\sin^31^\circ \cos^3\phi)}{6 \times (1-\tan^2\phi+e^{-2}\cos^2\phi) \times k_0 \times 10^{12}} ; \]
\[ A_6 = \frac{(p^6 \sin^61^\circ \times \sin\phi \times \cos^6\phi)}{720 \times (61-58\tan^2\phi+\tan^4\phi+270e^{-2}\cos^2\phi-330e^{-4}\sin^2\phi) \times k_0 \times 10^{24}} ; \]

and \[ B_5 = \frac{(p^5 \sin^51^\circ \times \cos^5\phi)}{120 \times (5-18\tan^2\phi+\tan^4\phi+14e^{-2}\cos^2\phi-58e^{-4}\sin^2\phi) \times k_0 \times 10^{20}} . \]

The static variables are:
\( \phi \) = latitude,
\( \theta \) = longitude,
\( a \) = semi-major axis of earth in projection being converted,
\( b \) = semi-minor axis,
\( k_0 = 0.9996 \),
\( e^2 = \text{eccentricity}^2 = (a^2-b^2) / a^2 \),
\( e'^2 = e^2 / 1-e^2 \),
\( r = \text{radius of curvature in central meridian} \)
\[ = a(1-e^2) / (1-e^2\sin^2\phi)^{3/2} \),
\[ v = \text{radius of curvature in prime vertical} \]
\[ = r(1 + e'^2 \cos^2 \phi), \]
\[ \Delta \theta = \theta_o - \theta \text{ (where } \theta_o = \text{longitude of central meridian)}, \]
\[ p = 0.0001 \Delta \theta \text{ (in seconds)}, \text{ and} \]
\[ S = \text{the meridional arc at the latitude in question.} \]

The value of S can be found in Department of the Army (1960), and changes continuously with latitude. In the Fortran program I wrote to perform the conversion to UTM coordinates, the meridional arc at 46.50° latitude is used as a constant. Over a limited latitude range this method introduces very little error.
program merc
   convert geographic coordinates (lat, lon) to UTM grid coords

   double precision phi, lambda, s, kzero, v, anom
   double precision a, b, esq, epsq, p, east, north, pi, dlambda
   double precision I, II, III, IV, V5, a6, b5
   double precision cosphi, sinphi, tanphi, onesec
   double precision k2, k3, k4, lon1, halfc

   phi = latitude
   lambda = longitude
   a = semi-major axis
   b = semi-minor axis
   s = meridional arc
   kzero = scale factor
   v = radius of curvature in prime vertical
   rho = radius of curvature in meridian
   esq = eccentricity squared

   using clark 1800 spheroid

   a = 6.378249145e+6
   b = a * (1. - (1. / 2.93465e+2))
   kzero = .9996
   pi = 3.14159265358979323846
   halfc = 180.
   onesec = (1. / 3600.) * pi / 180.
   esq = ((a**2.) - (b**2.)) / a**2.
   epsq = esq / (1. - esq)

61 format(f11.3, f12.3, f8.2)
open(unit=10, file='real', status='old', form='formatted')
read(10, *, end=500) lambda, phi, anomal

   call arccval(phi, s, v)
   lon1 = 117.0
   dlambda = abs(lon1 + lambda) * 3600.
   lambda = lambda * pi / halfc
   phi = phi * pi / halfc
   cosphi = cos(phi)
   sinphi = sin(phi)
   tanphi = tan(phi)

   rho = (a * (1. - esq)) / (1. - (esq * sinphi**2.))**1.5
   v = rho * (1. + (epsq * cosphi**2))
   p = .0001 * dlambda
   I = s * kzero

   call bigtwo(v, sinphi, cosphi, kzero, onesec, II)
   call bigthree(onesec, v, sinphi, cosphi, tanphi, epsq, kzero, III)
   call bigfour(v, cosphi, onesec, kzero, IV)
   call bigfive(v, onesec, cosphi, tanphi, epsq, kzero, V5)
   call asub6(p, v, onesec, sinphi, cosphi, tanphi, epsq, kzero, a6)
   call bsub5(p, onesec, v, cosphi, tanphi, epsq, sinphi, kzero, b5)

   north = I + (p**2. * II) + (III * p**4.) + a6
   east = (p * IV) + (p**3. * V5) + b5

   k2 = 1.4488e+0
   k3 = 5.000000e+5
   east = east / k2
   east = east + k3
   k4 = 9.99195e-1
   north = north / k4

   print *, 'I = ', I
   print *, 'II = ', II
   print *, 'III = ', III
   print *, 'IV = ', IV
   print *, 'V5 = ', V5
subroutine bigfive(v,onesec,cosphi,tanphi,epsq,kzero,V5)
double precision onesec,cosphi,tanphi,v
double precision epsq,kzero,V5,factor

V5 = ((sin(onesec)**3.) * v * cosphi**3.) / 6.
factor = (1. - (tanphi**2.) + (epsq * cosphi**2.))
print *, 'V5 = ',V5, ' factor = ',factor, 'kzero = ',kzero
V5 = V5 * factor * kzero * 1.0e+12

return
end

subroutine bigfour(v,cosphi,onesec,kzero,IV)
double precision v,cosphi,onesec,kzero,IV

IV = v * cosphi * sin(onesec) * kzero * 1.0e+4

return
end

subroutine bigthree(onesec,v,sinphi,cosphi,tanphi,epsq,kzero,III)
double precision onesec,v,sinphi,cosphi,tanphi
double precision epsq,kzero,III,factor

III = ((sin(onesec)**4.) * v * sinphi * cosphi**3.) / 24.
factor = 5.0 - tanphi**2. + (9.*epsq*(cosphi**2.)) +
1 (4. * (epsq**2.) * cosphi**4.)
III = III * factor * kzero * 1.0e+16

return
end

subroutine bigtwo (v,sinphi,cosphi,kzero,onesec,II)
double precision v,sinphi,cosphi,kzero,onesec,II

II = (v * sinphi * cosphi * (sin(onesec)**2.) /2.) *
1 kzero * 1.e+8

return
end
subroutine bsub5(p,onesec,v,cosphi,tanphi,epsq,sinphi,kzero,b5)
double precision p,onesec,v,cosphi,tanphi,epsq,sinphi,kzero,b5
double precision factor

b5 = (p**5.* (sin(onesec)**5.) * v * (cosphi**5.)) / 120.
factor = 5. - (18. * (tanphi**2.)) + tanphi**4. +
1 (14. * epsq * cosphi**2.) - (58. * epsq * sinphi**2.)
b5 = b5 * factor * kzero * 1.0e+20
return
end

subroutine asub6(p,v,onesec,sinphi,cosphi,tanphi,epsq,kzero,a6)
double precision p,onesec,sinphi,cosphi,tanphi,epsq,kzero,a6
double precision factor,v

a6 = (p**6.* (sin(onesec)**6.) * v * cosphi**5.) / 720.
factor = 61. - (58. * tanphi**2.) + tanphi**4. + (270.* epsq *
1 cosphi**2.) - (330. * epsq * sinphi**2.)
a6 = a6 * factor * kzero * 1.0e+24
return
end

subroutine arcval(lat,s,v)
double precision lat, s, v

lat = lat - 46.0
s = 5095773.677 + (lat * 111160.432)
v = 6389506.110 + (lat * 380.19)
c
print *, s
return
end

c print *,' a6 = ',a6
c print *,' b5= ',b5
write(6,61) east, north, anom
go to 100
500 stop
end
8. Appendix B: Origin and Testing of Computer Programs

MINC (Webring, 1981), FFTFIL (Hildenbrand, 1983), and SAKI (Webring, 1985), the three Fortran programs I used in this study, were obtained from the U.S.G.S. As received they were written to run under the Multics operating system and I made extensive modifications to all subroutine and input/output calls to enable them to run under the UNIX operating system on mainframe and minicomputers at the University of Montana. I also used Pro-Matlab (Moler et al., 1987) for fast Fourier transforms and filtering on the minicomputer to take full advantage of the computer’s graphics capabilities.

I used a test case to verify the reliability of the computer programs used in this study for gridding, fast Fourier transforms, filtering, and inverse modelling. First I wrote a Fortran program to generate a data set describing the Bouguer anomaly over a buried horizontal cylinder with negative density contrast (Figure 26), using the formula

\[ g = \frac{3.892 \rho R^2}{z} \frac{1}{(1 + x^2/z^2)} \]

where

- \( R \) = radius of buried cylinder in thousands of feet,
- \( z \) = depth to center of cylinder,
- \( x \) = horizontal distance to center of cylinder,
- \( \rho \) = density contrast in milligals (Dobrin, 1976).

I edited large and small blocks of data out of the set at pseudorandom spacings, to simulate the uneven spacing of field data, and tested MINC (Webring, 1981) with the edited data as input. The output grid conforms closely but not perfectly to the unedited data set.
FIGURE 26  Bouguer Anomaly Over A Buried Cylinder

Profile Across Row 32

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Next I calculated the Fourier transform of the grid with ProMatlab (Moler et al., 1987) and inverse transformed back to the state domain (Figure 28). The inverse transformed grid is identical to the grid created with MINC (Webring, 1981), as I expected. I constructed ramped band-pass filters to enhance or attenuate different frequency components of the grid (Figure 28), and their effects were as expected. Next I inverse modelled a two-dimensional profile from the output grid using SAKI (Webring, 1985). I started the program with a subsurface model close to the configuration of the original data set, and the model converged rapidly to conform with the original. After performing these tests and completing this study, I believe the reliability of the computer methods is adequate for a study using randomly spaced field data compiled by various investigators.
FIGURE 27  Inverse FFT of Test Case

Real Profile Across Row 32

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FIGURE 28 Frequency Domain RAMP Filter Operator: $k_1=16, k_2=32, k_3=44, k_4=63$

Buried Cylinder Grid RAMP Filtered & Inverted

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9. Appendix C: Mathematical Background

9.a: Gridding

In a geological study, where gravity anomalies are the additive result of many subsurface features at different scales, it is appropriate to construct a regularly spaced grid of gravity values over the study area. The uniform grid makes calculations more efficient and preserves observed values as closely as possible. The gridding algorithm I employed starts by constructing a very coarse grid. The grid point values are fixed by examining the observed values in the vicinity, and weighting their influence by their distance from the grid point. On subsequent iterations the distance between grid points is halved, and values of the previous grid are considered as well as observed values. MINC (Webring, 1981), the computer program used in this study, also saves the fourth previous grid as a template for filling in sparse regions. This means that the shortest wavelength present in sparse areas of the finished grid will be $4 \times$ grid spacing. A gridding algorithm must also ensure that as the distance between grid point and observation point approaches zero, the difference between grid value and observation approaches zero.

Interpolation to fill in a grid is two-dimensional cubic spline problem (Strang, 1986). Bouguer anomaly values over the study area can be considered as the displacement of the spline sheet. Along transects in the x direction, for each interval between two successive known values a
a cubic polynomial is calculated which connects the points (Briggs, 1974). MINC (Webring, 1983) also selects the cubics which minimize the overall curvature of the spline sheet, so this method probably provides the most geologically reasonable solutions. The cubic is the solution to
\[
\frac{\partial^4 u}{\partial x^4} + 2 \frac{\partial^2 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = f(n) \text{ at } (x_n, y_n)
\]
With four boundary conditions set by the values \(g_n\) and \(g_{n+1}\), and slopes \(s_n\) and \(s_{n+1}\) of the curve at the ends of the interval,
\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = s_n \text{ at } (x_n, y_n) \text{ and } \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = s_{n+1} \text{ at } (x_{n+1}, y_{n+1}).
\]
Any such cubic must have the form
\[
g = g_n(x-1)^2(2x+1) + s_n(x-1)^2x + g_{n+1}x^2(3-2x) + s_{n+1}x^2(x-1)
\]
(Strang, 1986). Interpolation along y-transects is analogous.

A piece-by-piece grid of cubic polynomials constructed in this way has a continuous second derivative, and the third derivatives are continuous between grid points, with a step at each grid point. This ensures the grid will be at least as smooth as, and typically much smoother than a surface defined by a single higher-order polynomial. High-order polynomials are subject to wild fluctuation between observation points (Strang, 1986). It can also be shown that any cubic solution minimizes the total curvature, \(C\), of the spline sheet, where
\[
C = \iint \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)^2 \, dx \, dy \quad \text{(Briggs, 1974; Strang, 1986).}
\]
9.b: Fourier Analysis and Synthesis

A grid of gravity values has x and y-axes with units of distance (in this case meters), and a z-axis with units of distance over time squared (milligals). A system where data changes with time and/or distance, such as gravity or groundwater flow on the Earth’s surface, is in the state domain (also called the space or time domain). In a very simple case, gravity might stay within a certain range of values and change regularly over distance, describing a waveform (Figure 29). The distance the waveform takes to cover one cycle, from one crest of the waveform to the next, is its wavelength. The number of cycles covered per unit distance is the waveform’s spatial frequency, so frequency = 1/wavelength. In this paper the term "frequency" refers to spatial frequency (also known as "wavenumber"). In Figure 29 the gravity signal has wavelength of twenty kilometers, frequency of 0.04 cycles per kilometer, and amplitude of one milligal. A grid of real gravity observations will not have such a simple form, but can be described as a combination of waveforms, each with different amplitudes and frequencies.

The Fourier transform allows representation of any state domain function as a combination of waveforms. The transformed data are in the frequency domain, where amplitude and phase vary with frequency. Each waveform of the function is described by pure harmonic functions $e^{ikx}$ or $\sin kx$ and $\cos kx$, where $k$ is frequency and $0 \leq x \leq 2\pi$. In this study the function is the Bouguer anomaly associated with an area of the earth’s surface. The value of the Fourier transform to a potential field
FIGURE 29 Idealized gravity anomaly waveform.
Wavelength = 20 km.
Spatial frequency = 0.05 cycles/km.
Amplitude = 1.0 milligal.
study is that transformed data can be filtered by multiplication, in the frequency domain, with the relevant filter's transform and then inverse transformed back. This saves considerable computation time over the alternative method of convolving data and filter (e.g., Byerly, 1965; Clement, 1973).

The mathematical literature on the Fourier transform is extensive (e.g., Bracewell, 1965). Strang (1986) presents the Fourier transform as a problem in matrix algebra. This approach is practical since that is how a digital computer must handle the Fourier transform.

For a function \( f(x) \), with \( 0 < x < 2\pi \), and infinitely many frequencies \( k \), the Fourier series of \( f \) is

\[
f(x) = \sum_{\infty} c_k e^{ikx} = a_0 + \sum_{1}^{\infty} (a_k \cos kx + b_k \sin kx)
\]

and

\[
a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \, dx ; \quad a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx \, dx ;
\]

\[
b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin kx \, dx ; \quad \text{and} \quad c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{ikx} \, dx .
\]

For an infinite continuous function \( f(x) \) with infinitely many frequencies \( k \), the Fourier integral representation of \( f \) is

\[
f(x) = \int_{-\infty}^{\infty} c(k) e^{ikx} \, dx .
\]

For a function \( f(x) \) on \( 0 \leq x \leq 2\pi \), with \( n \) specific frequencies \( k = 0,1,2,3,...,n-1 \), the discrete Fourier series is appropriate:

\[
f(x) = c_0 + c_1 e^{ix} + c_2 e^{2ix} + ... + c_{n-1} e^{i(n-1)x}
\]

(Cordell and Grauch, 1982).
In each case, the Fourier transform of $f(x)$ is the sequence of coefficients $a_k$ and $b_k$ or $c_k$. For use on a digital computer the discrete Fourier series is preferred because the coefficients are single numbers with real and complex parts (Hildenbrand, 1983). The Fourier coefficients $c_k$ are calculated by the equation

$$c_k = \frac{1}{n} \sum_{j=0}^{k} f(x) e^{-i2\pi jk/n}$$  \hspace{1cm} \text{(Strang, 1986)}.

The two forms of the Fourier series are not arbitrary expressions:

$$e^{ikx} = \cos kx + i \sin kx, \hspace{1cm} e^{-ikx} = \cos kx - i \sin kx;$$

so

$$c_k = 1/2a_k - 1/2ib_k, a_k = c_k + c_{k}^*, \hspace{1cm} b_k = 1/i c_k - c_{k}^*.$$  

Each Fourier coefficient $c_k$ is the "closest fit" in the least-squares sense of minimizing the error $E = \int_{-\pi}^{\pi} |f(x) - F(x)|^2 dx$.

Consider a signal $f$ as a vector with $n$ entries. To find the $n$ Fourier coefficients $c_k$ of $f$ (i.e., perform a Fourier analysis), one needs to find the vector $c$ such that $Fc = f$, where $F$ is a special $n \times n$ matrix. All the entries of $f$ are powers of $w$, where

$$w = e^{2\pi i/n},$$

an $n$th root of unity.

$F$ has the form

$$\begin{vmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & w & w^2 & \cdots & w^{(n-1)} \\
1 & w^2 & w^4 & \cdots & w^{2(n-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & w^{(n-1)} & w^{2(n-1)} & \cdots & w^{(n-1)^2}
\end{vmatrix}.$$  

Finding the Fourier coefficients of $f$ amounts to solving $c = F^{-1}f$.

There is a handy relationship due to $F$'s special form:

$$\overline{FF} = FF = nI$$

where $I$ is the identity matrix and $\overline{F}$ is the complex
conjugate of $F$, so that

$$F^{-1} = \frac{1}{n} \bar{F} \quad \text{and} \quad c = \frac{1}{n} \bar{F} f.$$

Fourier synthesis, finding the inverse Fourier transform of transformed data, is a matter of repeating the process outlined above. Fourier analysis transforms data from the state domain to the frequency domain, and Fourier synthesis does the reverse. The inverse Fourier transform of properly transformed data will be the original data.

The fast Fourier transform (FFT) reduces the number of computations needed to perform a Fourier analysis with an $n \times n$ Fourier matrix from $n^2$ to $1/2n \log_2 n$. This is possible because for all $n$ which are powers of two, all entries in the Fourier matrix $F$ are powers of $i$ (less time is saved for non-power of two matrices, but the method is still valid).

For example,

$$F_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}.$$

So $F_n$ is closely related to $F_{n/2}$, since for $n = 2m$, $w_n = w_m$. The matrix equation $f = F_c$ can be solved this way:

1) the vector $c$ is split into two vectors $c'$ and $c''$ where

$$c' = (c_0, c_2) \quad \text{and} \quad c'' = (c_1, c_3).$$

2) solve for $f' = F_2 c'$ and $f'' = F_2 c''$.

3) then compute the first two components of $f$ by

$$f_i = f'_i + w_i^n f''_i.$$
and compute the third and fourth components of $f$ by

$$f_{ji} = f_j - w^n f_j'. $$

These three steps perform multiplication of $F_4$ by $c$:

$$F_4 = \begin{bmatrix} 1 & 0 & 10 & |F_2| \\ 0 & 1 & 0 & i \\ 1 & 0 & -10 & |F_2| \\ 0 & 1 & 0 & -i \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Strang, 1986).
9.c: Digital Filtering

The simplest way to filter a two-dimensional data set such as a grid of gravity values is to multiply the Fourier transforms of the grid and filter (Hildenbrand, 1983). Mathematically:

\[ H(k,l) = F(k,l) \times G(k,l) , \]

where \( k \) and \( l \) are frequencies in the x and y directions, \( F(k,l) \) is the Fourier transform of the grid, \( G(k,l) \) is the transform of the filter, and \( H(k,l) \) is the filtered output. The output is then inverse transformed to the state domain to construct a filtered data set:

\[ h(x,y) = \sum_{k} \sum_{l} H(k,l) \exp\left(\frac{2\pi i (kx + ly)}{n}\right) . \]

For a given subsurface density distribution \( d(x',y',z') \), the gravity potential at \((x,y,z)\) above the distribution is

\[ g(x,y,z) = G \int \int \int \frac{d(x',y',z')}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} \, dx' \, dy' \, dz' . \]

where \( G \) in this case is the universal gravity constant. The Fourier transform of \( g(x,y,z) \) can then be considered in general as

\[ G(k,l,h) = 2\pi r \times p(k,l,h) \times H(k,l,h) , \]

where \( 2\pi r \) is a scale factor appropriate to the interval being studied, \( p(k,l,h) \) is a factor representing a density distribution, and \( H(k,l,h) \) is a factor representing the depth from measurement to source, with the form \( e^h (k^2 + l^2) \) (Gunn, 1975). Filtering any of these general factors is carried out by dividing the transformed grid by the factor's transform. Some filters enhance or suppress certain frequency components of the grid.
enhance or suppress certain frequency components of the grid.

Frequency targeted filters used in this study are second vertical
derivatives, upward and downward continuation, and band pass operators.

Vertical derivative filters enhance the higher frequencies in the grid,
without bracketing any specific range of frequencies. Their use is to
exaggerate closure around anomalies and make them appear more
clearly (Fuller, 1967; Hildenbrand, 1983). Without critical appraisal of the
filter operator in use and its results, vertical derivative filtering can create
anomalies in the output where none exist in the data (Fajklewicz, 1965).

For a potential field with single or multiple point sources, such as
gravity, Laplace’s equation holds and
\[
\frac{\partial^2 g(x,y,z)}{\partial z^2} = \frac{\partial^2 g(x,y,z)}{\partial x^2} - \frac{\partial^2 g(x,y,z)}{\partial y^2}
\]
for constant \(z\).

The Fourier transform of the second vertical derivative is
\[
G''(k,l) = 4\pi^2(k^2 + l^2)
\]
where \(k\) and \(l\) are the coordinates in the frequency domain (Fuller, 1967;
Strang, 1986). Calculating the second vertical derivative at each grid
point is done by multiplying each grid value by the operator \(4\pi^2(k^2+l^2)\). I
did not use third or higher vertical derivatives filters because the constant
term increases exponentially, increasing the amplitudes of false anomalies
(Fajklewicz, 1965).

Vertical continuation filters produce a simulation of the gravity field
measured at a datum above or below the elevation of the study area.
Downward continuation progressively enhances higher frequencies and exaggerates closure around anomalies until the depth of the anomaly's source is reached (Byerly, 1965; Fuller, 1967). At the depth of the source high frequency components increase tremendously in amplitude. Upward continuation (Figures 30, 31) enhances low frequency and suppresses high frequency components of the grid (Fuller, 1967; Hildenbrand, 1983). For two horizontal planes in a gravity field at heights z₁ and z₂, the gravity values at z₁ and z₂ are related by the Dirichlet integral

\[
g(x,y,z_2) = \frac{z_1 - z_2}{2\pi} \iint \frac{g(x',y',z_1)}{[(x-x')^2+(y-y')^2+(z_2-z_1)^2]^{3/2}} \, dx' \, dy'
\]

(Bhattacharyya, 1967). For continuation to a height h above the datum point (x,y,z=0),

\[
g(x,y,z=h) = \frac{h}{2\pi} \iint \frac{g(x,y,0)}{h^3} \, dx \, dy,
\]

and downward continuation is the reciprocal of upward continuation (Dean, 1958). The frequency domain operator for continuation is then

\[
G(k,l,h) = e^{2\pi h (k^2+l^2)},
\]

with positive h for downward continuation, negative for upward (Fuller, 1967).

Simple frequency domain operators can be constructed to delete a specific frequency or band of frequencies from the Fourier transform of the grid. The effect is to remove frequency components from the inverse transformed output. To delete frequencies between α and β from the...
FIGURE 30 Upward continuation filtered Bouguer anomaly map, northern Bitterroot valley. Contour interval 2 milligals. Upward continuation to 1.0 km. Scale 1:2500000. Compare to regional anomaly map (Fig. 8).
FIGURE 31 Rendered surface diagram, upward continuation of northern Bitterroot Bouguer anomalies. Continuation distance 1.0 km.
Fourier transform, and the associated frequency components from the
data set, one filter operator is:

\[ H(k,l) = 0 \text{ for } \alpha \leq k \text{ or } l \leq \beta \] , and
\[ = 1 \text{ for all other } k \text{ and } l \] .

An abrupt "square wave" operator such as the one above can introduce
spurious high frequency components to the output (Peterson and Dobrin,
1966; Dobrin, 1976). A ramped operator (Figure 28) which tapers up
from zero to one and then tapers back down produces smoother, more
reliable output (Hildenbrand, 1983). For \( a = \) the frequency where the
ramp begins, \( b = \) the frequency where the ramp assumes the value zero,
and \( c = \) the point where the ramp starts to rise back to one,

\[ H(k,l) = \frac{(k-a)}{(b-a)} \text{ for } a \leq k \leq b \text{ and } a \leq l \leq b \] ,
\[ = 0 \text{ for } b < k \leq c \text{ and } b < l \leq c \] ,
\[ = \frac{(d-k)}{(d-c)} \text{ for } c < k \leq d \text{ and } c \leq l \leq d \] ,
\[ = 1 \text{ elsewhere } \] .

Operators to pass a certain band of frequencies can be constructed using
the same relationships.

Directional trend filters either pass or reject azimuthal trends in the
gravity grid. A linear trend in the state domain is also a linear trend in
the frequency domain. If a trend has strike \( N\phi E \) where \( \phi = \tan^{-1}(dx/dy) \) ,
in the state domain, the "strike" in the frequency domain is \( N\theta E \) where
\[ \theta = \tan^{-1}(dk/dl) \text{ and } dk/dl = -dx/dy \] (Fuller, 1967). I constructed a "pie-slice" frequency domain filter to reject
or pass trends within a specified angle of a given strike \( \theta \). Note that as
the "slice" widens towards the edge of the grid, a wider band around the chosen strike is removed from the higher frequencies.
10. Appendix D: Gravity Observations.

These are the Bouguer anomaly values used in this study, listed with their UTM coordinates. The leftmost column is northing, the center column is easting, and the right column is observed Bouguer anomaly. Eastings greater than 600000 are in Zone 11T, and eastings less than 300000 are in Zone 12T. For gridding purposes I eventually converted all the easting values to Zone 11T. The seam between the zones runs straight through the middle of the Bitterroot valley.
FIGURE 32 Distribution of observations used in study. Coordinates in UTM projection, zone 11T. * = observation location.

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